TECHNICAL APPENDIX I: SOLUTION ALGORITHM AND ROBUSTNESS

(Not for Publication)

Establishment Heterogeneity, Exporter Dynamics, and the Effects of Trade Liberalization*

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These notes are divided in the following sections and subsections.

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1. Equations for the Model Solution

The following equations describe the complete economy.

A. Consumer's Problem

Home consumers choose consumption, investment, and bond holdings to maximize their utility:

$$V_{C,0} = \max \sum_{t=0}^{\infty} \beta^t U\left(C_t\right), \tag{1}$$

subject to the sequence of budget constraints,

$$P_t C_t + P_t K_t + Q_t B_t \leq P_t W_t L_t + P_t R_t K_{t-1} + \left(1-\delta\right) P_t K_{t-1} + B_{t-1} + P_t \Pi_t + P_t T_t, \tag{2}$$

where β is the subjective time discount factor with $0 < \beta < 1$; P_t is the price of the final good; C_t is the consumption of final goods; K_{t-1} is the capital available in period t; Q_t and B_t are the price of bonds and the bond holdings; W_t and R_t denote the real wage rate and the rental rate of capital; δ is the depreciation rate of capital; Π_t is the sum of real dividends from the home country producers; and T_t is the real lump-sum transfer from the home government.

The problem of foreign consumers is analogous to this problem. Prices and allocations in the foreign country are represented with an asterisk. Money has no role in this economy and is only a unit of account. The foreign budget constraint is expressed as

$$P_t^* C_t^* + P_t^* K_t^* + \frac{Q_t}{e_t} B_t^* \leq P_t^* W_t^* L_t^* + P_t^* R_t^* K_{t-1}^* + \left(1-\delta\right) P_t^* K_{t-1}^* + \frac{B_{t-1}^*}{e_t} + P_t^* \Pi_t^* + P_t^* T_t^*, \tag{3}$$

where * denotes the foreign variables and e_t is the nominal exchange rate with home currency as numeraire.[1]

The first-order conditions for the home consumers are:

$$Q_t = \beta \frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}}, \tag{4}$$

$$1 = \beta \frac{U_{C,t+1}}{U_{C,t}} \left(R_{t+1} + 1 - \delta\right), \tag{5}$$

[1] An increase in e_t means a depreciation of domestic currency.

where $U_{C,t} = \partial U(C_t)/\partial C_t$. Similarly, the first-order conditions for the foreign consumers are:

$$Q_t = \beta \frac{U^*_{C,t+1}}{U^*_{C,t}} \frac{e_t P^*_t}{e_{t+1} P^*_{t+1}}, \tag{6}$$

$$1 = \beta \frac{U^*_{C,t+1}}{U^*_{C,t}} \left(R^*_{t+1} + 1 - \delta\right). \tag{7}$$

From the bond equations (4) and (6), we get

$$\frac{U_{C,t+1} P_t}{U_{C,t} P_{t+1}} = \frac{U^*_{C,t+1} P^*_t e_t}{U^*_{C,t} P^*_{t+1} e_{t+1}}. \tag{8}$$

The real exchange rate is defined as $q_t = e_t P^*_t / P_t$. Iterating on (8) yields

$$q_t = \kappa_0 \frac{U^*_{C,t}}{U_{C,t}}, \tag{9}$$

where $\kappa_0 = q_0 U_{C,0}/U^*_{C,0}$.

B. Final Good Producer's Problem

The production technology of the final good producer is given by a Cobb-Douglas function for tradable and non-tradable aggregate inputs, $D_{T,t}$ and $D_{N,t}$, with the tradable share γ

$$D_t = D^{\gamma}_{T,t} D^{1-\gamma}_{N,t}, \tag{10}$$

where D_t is the output of final goods and $D_{T,t}$ and $D_{N,t}$ are the aggregates of tradable and non-tradable goods, respectively. The aggregation technology of the establishment is given by a constant elasticity of substitution (henceforth CES) function

$$D_{T,t} = \left[\sum_{m=0}^{1} \int_v \int_z y^d_{H,t}(z,v,m)^{\frac{\theta-1}{\theta}} \varphi_{T,t}(z,v,m)\, dz dv + \int_v \int_z y^d_{F,t}(z,v,1)^{\frac{\theta-1}{\theta}} \varphi^*_{T,t}(z,v,1)\, dz dv \right]^{\frac{\theta}{\theta-1}}, \tag{11}$$

$$D_{N,t} = \left(\int_z y^d_{N,t}(z)^{\frac{\theta-1}{\theta}} \varphi_{N,t}(z)\, dz \right)^{\frac{\theta}{\theta-1}}, \tag{12}$$

where $\varphi_{T,t}(z,v,m)$ and $\varphi^*_{T,t}(z,v,m)$ are the measure of home and foreign country tradable establishments with technology z, exporting fixed cost shock v, and export status, $m=1$ for exporters and $m=0$ for non-exporters, respectively; $\varphi_{N,t}(z)$ is the measure of home country non-tradable establishments with technology z; $y^d_{H,t}(z,v,m)$, $y^d_{F,t}(z,v,1)$, and $y^d_{N,t}(z)$ are inputs of intermediate goods purchased from a home tradable good producer with technology z, fixed cost shock v, and export status m, foreign tradable exporter with technology z and fixed cost shock v, and home non-tradable good producer with technology z, respectively. The elasticity of substitution between intermediate goods within a sector is θ. The final goods market is perfectly competitive. Given the final good price at home P_t and the prices charged for each type of tradable and non-tradable good the final good producer solves the following problem

$$\begin{aligned}\max \Pi_{F,t} &= D_t - \sum_{m=0}^{1}\int_v\int_z \left[\frac{P_{H,t}(z,v,m)}{P_t}\right] y^d_{H,t}(z,v,m)\,\varphi_{T,t}(z,v,m)\,dzdv \\ &\quad -\int_v\int_z \left[\frac{(1+\tau)P_{F,t}(z,v,1)}{P_t}\right] y^d_{F,t}(z,v,1)\varphi^*_{T,t}(z,v,1)\,dzdv \\ &\quad -\int_z \left[\frac{P_{N,t}(z)}{P_t}\right] y^d_{N,t}(z)\varphi_{N,t}(z)\,dz,\end{aligned} \tag{13}$$

subject to the production technology (10), (11), (12).[2] Here, $P_{H,t}(z,v,m)$, $P_{F,t}(z,v,1)$, and $P_{N,t}(z)$ are the prices of intermediated goods produced by home tradable good producer with (z,v,m), foreign tradable good producers with $(z,v,1)$, and the home non-tradable good producers with z, respectively. The variable τ is the ad valorem tariff rate. The first-order conditions for the home final good producer give the input demand functions,

$$y^d_{H,t}(z,v,m) = \gamma\left[\frac{P_{H,t}(z,v,m)}{P_t}\right]^{-\theta}\left(\frac{P_{T,t}}{P_t}\right)^{\theta-1} D_t, \tag{14}$$

$$y^d_{F,t}(z,v,1) = \gamma\left[\frac{(1+\tau)P_{F,t}(z,v,1)}{P_t}\right]^{-\theta}\left(\frac{P_{T,t}}{P_t}\right)^{\theta-1} D_t, \tag{15}$$

$$y^d_{N,t}(z) = (1-\gamma)\left[\frac{P_{N,t}(z)}{P_t}\right]^{-\theta}\left(\frac{P_{N,t}}{P_t}\right)^{\theta-1} D_t, \tag{16}$$

[2] Note that the production function is defined only over the available products. It is equivalent to define the production function over all possible varieties but constrain purchases of some varieties to be zero.

where the price indices are defined as

$$P_{T,t} = \left\{ \sum_{m=0}^{1} \int_v \int_z P_{H,t}(z,v,m)^{1-\theta} \varphi_{T,t}(z,v,m)\, dzdv \right. \tag{17}$$

$$\left. + \int_v \int_z [(1+\tau) P_{F,t}(z,v,1)]^{1-\theta} \varphi^*_{T,t}(z,v,1)\, dzdv \right\}^{\frac{1}{1-\theta}},$$

$$P_{N,t} = \left[\int_z P_{N,t}(z)^{1-\theta} \varphi_{N,t}(z)\, dz \right]^{\frac{1}{1-\theta}}, \tag{18}$$

$$P_t = \left(\frac{P_{T,t}}{\gamma} \right)^{\gamma} \left(\frac{P_{N,t}}{1-\gamma} \right)^{1-\gamma}. \tag{19}$$

Final goods are used for consumption and investment. The final good resource constraint is

$$D_t = C_t + I_t, \tag{20}$$

and investment, I_t, is given by the law of motion for capital,

$$I_t = K_t - (1-\delta) K_{t-1}. \tag{21}$$

C. Intermediate Good Producers

All the intermediate good producers produce their differentiated goods using capital and labor. Tradable good producers also use material inputs produced by other tradable good producers. We assume that an incumbent's productivity, z, follows a first order Markov process with a transition probability $\phi(z'|z)$, the probability that the productivity of the establishment will be z' in the next period conditional on its current productivity, z, provided that the establishment survived, with z, $z' \in (\underline{z}, \overline{z})$. A tradable good producer draws its exporting fixed cost shock each period from $\phi_v(v)$. An entrant draws productivity next period from $\phi_E(z')$. We also assume that establishments receive an exogenous death shock that depends on its productivity, z, at the end of the period, $0 \leq n_d(z) \equiv 1 - n_s(z) \leq 1$.

Non-Tradable Good Producers' Problem

A producer with technology z in the non-tradable good sector chooses its current price $P_{N,t}(z)$, inputs of labor $l_{N,t}(z)$ and capital $k_{N,t}(z)$ given a Cobb-Douglas production technology,

$$y_{N,t}(z) = e^z k_{N,t}(z)^{\alpha} l_{N,t}(z)^{1-\alpha}, \tag{22}$$

to solve

$$V_{N,t}(z) = \max \Pi_{N,t}(z) + n_s(z) Q_t \left(\frac{P_{t+1}}{P_t}\right) \int_{z'} V_{N,t+1}(z') \phi(z'|z) dz', \tag{23}$$

$$\Pi_{N,t}(z) = \left[\frac{P_{N,t}(z)}{P_t}\right] y_{N,t}(z) - W_t l_{N,t}(z) - R_t k_{N,t}(z), \tag{24}$$

subject to the production technology (22), and the constraints that supplies to the non-tradable goods market $y_{N,t}(z)$ are equal to demands by final good producers $y^d_{N,t}(z)$ in (16).

The first-order conditions for a non-tradable good producer from home country with technology z give the price decision rule

$$\frac{P_{N,t}(z)}{P_t} = \left(\frac{\theta}{\theta-1}\right) e^{-z} MC_{N,t}, \tag{25}$$

$$MC_{N,t} = \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}, \tag{26}$$

where $MC_{N,t}$ is the marginal cost of production for a non-tradable good producer with $z = 0$, and the demand for capital and labor are

$$k_{N,t}(z) = \alpha e^{-z} \left(\frac{MC_{N,t}}{R_t}\right) y_{N,t}(z), \tag{27}$$

$$l_{N,t}(z) = (1-\alpha) e^{-z} \left(\frac{MC_{N,t}}{W_t}\right) y_{N,t}(z). \tag{28}$$

From the production technology (22), the constraints that supplies to the non-tradable goods market $y_{N,t}(z)$ are equal to demands by final good producers $y^d_{N,t}(z)$ in (16), and the first-order conditions, the labor and capital demand functions can be obtained as

$$k_{N,t}(z) = (1-\gamma) \left(\frac{\theta-1}{\theta}\right) \left(\frac{\theta MC_{N,t}}{\theta-1}\right)^{1-\theta} e^{(\theta-1)z} \left(\frac{\alpha}{R_t}\right) \left(\frac{P_{N,t}}{P_t}\right)^{\theta-1} D_t. \tag{29}$$

$$l_{N,t}(z) = (1-\gamma) \left(\frac{\theta-1}{\theta}\right) \left(\frac{\theta MC_{N,t}}{\theta-1}\right)^{1-\theta} e^{(\theta-1)z} \left(\frac{1-\alpha}{W_t}\right) \left(\frac{P_{N,t}}{P_t}\right)^{\theta-1} D_t, \tag{30}$$

The output of a non-tradable good producer with technology z is then obtained as

$$y_{N,t}(z) = (1-\gamma) \left(\frac{\theta MC_{N,t}}{\theta-1}\right)^{-\theta} e^{\theta z} \left(\frac{P_{N,t}}{P_t}\right)^{\theta-1} D_t. \tag{31}$$

The maximized real profit and the value of the non-tradable good producer are given as

$$\Pi_{N,t}(z) = \left(\frac{1-\gamma}{\theta}\right)\left(\frac{\theta MC_{N,t}}{\theta-1}\right)^{1-\theta} e^{(\theta-1)z}\left(\frac{P_{N,t}}{P_t}\right)^{\theta-1} D_t, \tag{32}$$

$$V_{N,t}(z) = \Pi_{N,t}(z) + n_s(z) Q_t \left(\frac{P_{t+1}}{P_t}\right)\int_{z'} V_{N,t+1}(z')\,\phi(z'|z)\,dz'. \tag{33}$$

With constant mark-up pricing in (25) and the price index of the non-tradable aggregate in (18) we have

$$\begin{aligned}\left(\frac{P_{N,t}}{P_t}\right)^{1-\theta} &= \int_z \left[\frac{P_{N,t}(z)}{P_t}\right]^{1-\theta} \varphi_{N,t}(z)\,dz \\ &= \left(\frac{\theta MC_t}{\theta-1}\right)^{1-\theta}\Psi_{N,t},\end{aligned} \tag{34}$$

where $\Psi_{N,t} = \int_z e^{(\theta-1)z}\varphi_{N,t}(z)\,dz$. Aggregate demand for labor and capital by non-tradable good producers is

$$L_{N,t} = \int_z l_{N,t}(z)\,\varphi_{N,t}(z)\,dz = (1-\gamma)\left(\frac{\theta-1}{\theta}\right)\left(\frac{1-\alpha}{W_t}\right)D_t, \tag{35}$$

$$K_{N,t} = \int_z k_{N,t}(z)\,\varphi_{N,t}(z)\,dz = (1-\gamma)\left(\frac{\theta-1}{\theta}\right)\left(\frac{\alpha}{R_t}\right)D_t. \tag{36}$$

The expected value of an entrant is given as

$$V^E_{N,t} = -W_t f_E + Q_t\left(\frac{P_{t+1}}{P_t}\right)\int_{z'} V_{N,t+1}(z')\,\phi_E(z')\,dz', \tag{37}$$

which is equal to zero in equilibrium. Here, f_E is the entry cost.

The measure of non-tradable good producers with productivity z evolves as

$$\varphi_{N,t+1}(z') = \int_z n_s(z)\,\varphi_{N,t}(z)\,\phi(z'|z)\,dz + N_{NE,t}\phi_E(z'), \tag{38}$$

where $N_{NE,t}$ is the mass of entrants in the non-tradable good sector that pay the entry cost in period t, and the mass of establishments in the non-tradable good sector is written as

$$N_{N,t} = \int_z \varphi_{N,t}(z)\,dz. \tag{39}$$

Tradable Good Producers' Problem

A producer in the tradable good sector is described by its technology, exporting fixed cost shock, and export status, (z, v, m). Each period, it chooses current prices for home and foreign markets, $P_{H,t}(z, v, m)$ and $P^*_{H,t}(z, v, m)$; inputs of labor $l_{T,t}(z, v, m)$, capital $k_{T,t}(z, v, m)$, and materials $x_t(z, v, m)$; and next period's export status, m'. Total materials, $x_t(z, v, m)$, is composed of tradable intermediate goods with a constant elasticity of substitution function

$$\begin{aligned} x_t(z,v,m) \quad = \quad & \left[\sum_{\mu=0}^{1}\int_{\varpi}\int_{\zeta} x^d_{H,t}(\zeta,\varpi,\mu,z,v,m)^{\frac{\theta-1}{\theta}}\varphi_{T,t}(\zeta,\varpi,\mu)\,d\zeta d\varpi \right. \\ & \left. +\int_{\varpi}\int_{\zeta} x^d_{F,t}(\zeta,\varpi,1,z,v,m)^{\frac{\theta-1}{\theta}}\varphi^*_{T,t}(\zeta,\varpi,1)\,d\zeta d\varpi\right]^{\frac{\theta}{\theta-1}}, \end{aligned} \tag{40}$$

where $x^d_{H,t}(\zeta, \varpi, \mu, z, v, m)$ and $x^d_{F,t}(\zeta, \varpi, 1, z, v, m)$ are inputs of intermediate goods purchased from a home tradable good producer with technology ζ, fixed cost shock ϖ, and export status μ, and a foreign tradable exporter with technology ζ and fixed cost shock ϖ, respectively, by the tradable good producer with technology z, fixed cost shock v, and export status m. The CES aggregation function gives the input demand functions,

$$x^d_{H,t}(\zeta,\varpi,\mu,z,v,m) \quad = \quad \left[\frac{P_{H,t}(\zeta,\varpi,\mu)}{P_t}\right]^{-\theta}\left(\frac{P_{T,t}}{P_t}\right)^{\theta} x_t(z,v,m), \tag{41}$$

$$x^d_{F,t}(\zeta,\varpi,1,z,v,m) \quad = \quad \left[\frac{(1+\tau)P_{F,t}(\zeta,\varpi,1)}{P_t}\right]^{-\theta}\left(\frac{P_{T,t}}{P_t}\right)^{\theta} x_t(z,v,m), \tag{42}$$

given the prices and the choice of the aggregate material input, $x_t(z, v, m)$.

The producer has a Cobb-Douglas production technology,

$$y_{T,t}(z,v,m) = e^z\left[k_{T,t}(z,v,m)^{\alpha}\, l_{T,t}(z,v,m)^{1-\alpha}\right]^{1-\alpha_x} x(z,v,m)^{\alpha_x}, \tag{43}$$

and solves

$$
\begin{aligned}
V_{T,t}\left(z,v,m\right) &= \max \Pi_{T,t}\left(z,v,m\right) - m' W_t e^{v}\left[f_1 m + (1-m) f_0\right] \\
&\quad + n_s\left(z\right) Q_t \left(\frac{P_{t+1}}{P_t}\right) \int_{v'} \int_{z'} V_{T,t}\left(z',v',m'\right) \phi\left(z'|z\right) \phi_v\left(v'\right) dz' dv', \qquad (44)\\
\Pi_{T,t}\left(z,v,m\right) &= \left[\frac{P_{H,t}\left(z,v,m\right)}{P_t}\right] y_{H,t}\left(z,v,m\right) + m\left[\frac{e_t P^*_{H,t}\left(z,v,m\right)}{P_t}\right] y^*_{H,t}\left(z,v,m\right) \qquad (45)\\
&\quad - W_t l_{T,t}\left(z,v,m\right) - R_t k_{T,t}\left(z,v,m\right) \\
&\quad - \sum_{\mu=0}^{1} \int_{\varpi} \int_{\zeta} \left[\frac{P_{H,t}\left(\zeta,\varpi,\mu\right)}{P_t}\right] x^d_{H,t}\left(\zeta,\varpi,\mu,z,v,m\right) \varphi_{T,t}\left(\zeta,\varpi,\mu\right) d\zeta d\varpi \\
&\quad - \int_{\varpi} \int_{\zeta} \left[\frac{(1+\tau) P_{F,t}\left(\zeta,\varpi,1\right)}{P_t}\right] x^d_{F,t}(\zeta,\varpi,1,z,v,m) \varphi^*_{T,t}\left(\zeta,\varpi,1\right) d\zeta d\varpi,
\end{aligned}
$$

subject to the production technology (43) and the constraints that supplies to home and foreign tradable goods markets, $y_{H,t}\left(z,v,m\right)$ and $y^*_{H,t}\left(z,v,m\right)$ with $y_{T,t}\left(z,v,m\right) = y_{H,t}\left(z,v,m\right) + (1+\xi)\, y^*_{H,t}\left(z,v,m\right)$, are equal to demands by final good producers from (14), the foreign analogue of (15),

$$
y^{d*}_{H,t}\left(z,v,m\right) = m\gamma \left[\frac{(1+\tau) P^*_{H,t}\left(z,v,m\right)}{P^*_t}\right]^{-\theta} \left(\frac{P^*_{T,t}}{P^*_t}\right)^{\theta-1} D^*_t, \qquad (46)
$$

and demands by intermediate good producers

$$
\sum_{\mu=0}^{1} \int_{\zeta} \int_{\varpi} x^d_{H,t}\left(z,v,m,\zeta,\varpi,\mu\right) \varphi_{T,t}\left(\zeta,\varpi,\mu\right) d\varpi d\zeta, \qquad (47)
$$

$$
m \sum_{\mu=0}^{1} \int_{\zeta} \int_{\varpi} x^{d*}_{H,t}\left(z,v,m,\zeta,\varpi,\mu\right) \varphi^*_{T,t}\left(\zeta,\varpi,\mu\right) d\varpi d\zeta, \qquad (48)
$$

with

$$
\begin{aligned}
y_{H,t}\left(z,v,m\right) &= y^d_{H,t}\left(z,v,m\right) + \sum_{\mu=0}^{1} \int_{\varpi} \int_{\zeta} x^d_{H,t}\left(z,v,m,\zeta,\varpi,\mu\right) \varphi_{T,t}\left(\zeta,\varpi,\mu\right) d\zeta d\varpi, \qquad (49)\\
y^*_{H,t}\left(z,v,m\right) &= y^{d*}_{H,t}\left(z,v,m\right) + m \sum_{\mu^*=0}^{1} \int_{\varpi} \int_{\zeta} x^{d*}_{H,t}\left(z,v,m,\zeta,\varpi,\mu\right) \varphi^*_{T,t}\left(\zeta,\varpi,\mu\right) d\zeta d\varpi. \qquad (50)
\end{aligned}
$$

Let the value of the producer with (z, v, m) if it decides to export in period $t+1$ be

$$\begin{aligned} V_{T,t}^{1}(z,v,m) &= \max \Pi_{T,t}(z,v,m) - W_t e^{v}\left[f_1 m + (1-m) f_0\right] \\ &+ n_s(z)\, Q_t \left(\frac{P_{t+1}}{P_t}\right) \int_{v'} \int_{z'} V_{T,t+1}\left(z',v',1\right) \phi\left(z'|z\right) \phi_v\left(v'\right) dz' dv', \end{aligned} \tag{51}$$

and let the value if it decides not to export in period $t+1$ be

$$\begin{aligned} V_{T,t}^{0}(z,v,m) &= \max \Pi_{T,t}(z,v,m) \\ &+ n_s(z)\, Q_t \left(\frac{P_{t+1}}{P_t}\right) \int_{v'} \int_{z'} V_{T,t+1}\left(z',v',0\right) \phi\left(z'|z\right) \phi_v\left(v'\right) dz' dv'. \end{aligned} \tag{52}$$

Then, the actual value of the producer can be defined as

$$\begin{aligned} V_{T,t}(z,v,m) &= \max\left\{V_{T,t}^{1}(z,v,m), V_{T,t}^{0}(z,v,m)\right\} \\ &= \max \Pi_{T,t}(z,v,m) + \max\left\{-W_t e^{v}\left[f_1 m + (1-m) f_0\right]\right. \\ &\quad + n_s(z)\, Q_t \left(\frac{P_{t+1}}{P_t}\right) \int_{v'} \int_{z'} V_{T,t+1}\left(z',v',1\right) \phi\left(z'|z\right) \phi_v\left(v'\right) dz' dv', \\ &\quad \left. n_s(z)\, Q_t \left(\frac{P_{t+1}}{P_t}\right) \int_{v'} \int_{z'} V_{T,t+1}\left(z',v',0\right) \phi\left(z'|z\right) \phi_v\left(v'\right) dz' dv' \right\}. \end{aligned} \tag{53}$$

Clearly the value of a producer depends on its export status and fixed cost shock, and is monotonically increasing and continuous in z given m, v, and the states of the world. Moreover V_T^1 intersects V_T^0 from below as long as there are some establishments that do not export.[3] Hence, it is possible to solve for the establishment productivity at which an establishment is indifferent between exporting and not exporting; that is, the increase in establishment value from exporting equals the cost of exporting. This level of establishment productivity differs by the establishment's current export status and fixed cost shock. The critical level of technology for exporters and non-exporters $z_{m,t}(v)$ satisfies

$$V_{T,t}^{1}\left(z_{m,t}(v), v, m\right) = V_{T,t}^{0}\left(z_{m,t}(v), v, m\right) \tag{54}$$

if there exist $z_{m,t}^{*}(v)$ and $z_{m,t}^{**}(v)$ such that $V_{T,t}^{1}\left(z_{m,t}^{*}(v), v, m\right) < V_{T,t}^{0}\left(z_{m,t}^{*}(v), v, m\right)$ and $V_{T,t}^{1}\left(z_{m,t}^{**}(v), v, m\right)$ $> V_{T,t}^{0}\left(z_{m,t}^{**}, v, m\right)$. If $V_{T,t}^{1}(z,v,m) < V_{T,t}^{0}(z,v,m)$ for all $z \in (\underline{z},\ \overline{z})$, all the producers with v will not pay the fixed cost. In that case, we set $z_{m,t}(v) = \overline{z}$.[4] If $V_{T,t}^{1}(z,v,m) > V_{T,t}^{0}(z,v,m)$ for all $z \in (\underline{z},\ \overline{z})$,

[3] If the difference between f_0 and f_1 is relatively large, the economy may have $V^1 > V^0$ for all $z \in (\underline{z},\ \overline{z})$ for some states of the world.

[4] Note that if $\overline{z} = \infty$, for any v, there exits $\widetilde{z} \in (\underline{z},\ \overline{z})$ such that $V_{T,t}^{1}(\widetilde{z}, v, m) = V_{T,t}^{0}(\widetilde{z}, v, m)$ if there exists $z^{*} \in (\underline{z},\ \overline{z})$ such that $V_{T,t}^{1}(z^{*}, v, m) < V_{T,t}^{0}(z^{*}, v, m)$.

all the producers with v pay the fixed cost. In that case, we set $z_{m,t}(v) = \underline{z}$.

The first-order conditions for a non-tradable good producer from home country with technology z, fixed cost shock v, and export status m give the prices for home and foreign as

$$\frac{P_{H,t}(z,v,m)}{P_t} = \left(\frac{\theta}{\theta-1}\right) e^{-z} MC_{T,t}, \tag{55}$$

$$\frac{P^*_{H,t}(z,v,1)}{P^*_t} = (1+\xi)\left(\frac{\theta}{\theta-1}\right) q_t^{-1} e^{-z} MC_{T,t}, \tag{56}$$

$$MC_{T,t} = \left(\frac{P_{T,t}/P_t}{\alpha_x}\right)^{\alpha_x} \left(\frac{MC_{N,t}}{1-\alpha_x}\right)^{1-\alpha_x}, \tag{57}$$

where $MC_{T,t}$ is the marginal cost of production for a tradable good producer with $z = 0$, and the demands for capital, labor, and material inputs

$$k_{T,t}(z,v,m) = \alpha(1-\alpha_x) e^{-z} MC_{T,t}\left[\frac{y_{T,t}(z,v,m)}{R_t}\right], \tag{58}$$

$$l_{T,t}(z,v,m) = (1-\alpha)(1-\alpha_x) e^{-z} MC_{T,t}\left[\frac{y_{T,t}(z,v,m)}{W_t}\right], \tag{59}$$

$$x_t(z,v,m) = \alpha_x e^{-z} MC_{T,t}\left[\frac{y_{T,t}(z,v,m)}{P_{T,t}/P_t}\right]. \tag{60}$$

In equilibrium, the total output equals to the sum of demand by final good producers and intermediate good producers in the tradable good sector,

$$\begin{aligned} y_{T,t}(z,v,m) &= y^d_{H,t}(z,v,m) + \sum_{\mu=0}^{1}\int_{\varpi}\int_{\zeta} x_{H,t}(\zeta,\varpi,\mu,z,v,m)\,\varphi_{T,t}(\zeta,\varpi,\mu)\,d\zeta d\varpi \\ &\quad + (1+\xi)\left\{ y^{d*}_{H,t}(z,v,m) + m\sum_{\mu=0}^{1}\int_{\varpi}\int_{\zeta} x_{F,t}(\zeta,\varpi,\mu,z,v,m)\varphi^*_{T,t}(\zeta,\varpi,\mu)\,d\zeta d\varpi \right\}. \end{aligned} \tag{61}$$

From the price decisions in (55) and the foreign analog of (56), the price index for the aggregate tradable goods is given as

$$\frac{P_{T,t}}{P_t} = \left\{ \left(\frac{\theta MC_{T,t}}{\theta-1}\right)^{1-\theta} \Psi_{T,t} + \left[\frac{\theta(1+\tau)(1+\xi)\, q_t MC^*_{T,t}}{\theta-1}\right]^{1-\theta} \Psi^*_{X,t} \right\}^{\frac{1}{1-\theta}}, \tag{62}$$

where $\Psi_{T,t} = \sum_{m=0}^{1}\int_v\int_z e^{(\theta-1)z}\varphi_{T,t}(z,v,m)\,dzdv$ and $\Psi^*_{X,t} = \int_v\int_z e^{(\theta-1)z}\varphi^*_{T,t}(z,v,1)\,dzdv$. From the demand functions of tradable goods we have

$$y_{T,t}(z,v,m) = \left(\frac{\theta MC_T}{\theta-1}\right)^{-\theta} e^{\theta z}\left(\frac{P_{T,t}}{P_t}\right)^{\theta-1}(\gamma D_t + \alpha_x MC_{T,t}H_t) \\ + m(1+\xi)^{1-\theta}\left[\frac{(1+\tau)\theta MC_{T,t}}{\theta-1}\right]^{-\theta} e^{\theta z} q_t^{\theta}\left(\frac{P^*_{T,t}}{P^*_t}\right)^{\theta-1}\left(\gamma D^*_t + \alpha_x MC^*_{T,t}H^*_t\right), \tag{63}$$

where

$$H_t = \sum_{m=0}^{1}\int_v\int_z e^{-z} y_{T,t}(z,v,m)\,\varphi_{T,t}(z,v,m)\,dzdv, \tag{64}$$

$$H^*_t = \sum_{m=0}^{1}\int_v\int_z e^{-z} y^*_{T,t}(z,v,m)\,\varphi^*_{T,t}(z,v,m)\,dzdv. \tag{65}$$

By multiplying both sides by e^{-z} in (63) and integrating over producers, we have

$$\begin{aligned} H_t &\equiv \sum_{m=0}^{1}\int_v\int_z e^{-z} y_{T,t}(z,v,m)\,\varphi_{T,t}(z,v,m)\,dzdv \\ &= \left(\frac{\theta MC_T}{\theta-1}\right)^{-\theta}\left(\frac{P_{T,t}}{P_t}\right)^{\theta-1}\Psi_{T,t}(\gamma D_t + \alpha_x MC_{T,t}H_t) \\ &\quad + (1+\xi)^{1-\theta}(1+\tau)^{-\theta} q_t^{\theta}\left(\frac{\theta MC_T}{\theta-1}\right)^{-\theta}\left(\frac{P^*_{T,t}}{P^*_t}\right)^{\theta-1}\Psi_{X,t}\left(\gamma D^*_t + \alpha_x MC^*_{T,t}H^*_t\right). \end{aligned} \tag{66}$$

From the demands for capital, labor, and material inputs in (58), (59) and (60) together with (63) and (66), we have the aggregate demands for labor and capital in the tradable good sector as

$$\begin{aligned} L_{T,t} &= \sum_{m=0}^{1}\int_v\int_z l_{T,t}(z,v,m)\,\varphi_{T,t}(z,v,m)\,dzdv \\ &= (1-\alpha)(1-\alpha_x)\frac{MC_{T,t}}{W_t}H_t, \end{aligned} \tag{67}$$

$$\begin{aligned} K_{T,t} &= \sum_{m=0}^{1}\int_v\int_z l_{T,t}(z,v,m)\,\varphi_{T,t}(z,v,m)\,dzdv \\ &= \alpha(1-\alpha_x)\frac{MC_{T,t}}{R_t}H_t. \end{aligned} \tag{68}$$

From the price decisions in (55) and (56), and the equilibrium output level in (63) the maximized real profit of a firm is given as

$$\begin{aligned}\Pi_{T,t}(z,v,m) &= \left(\frac{1}{\theta-1}\right) MC_{T,t}e^{-z}y_{T,t}(z,v,m) \qquad (69)\\ &= \frac{1}{\theta}\left(\frac{\theta MC_T}{\theta-1}\right)^{1-\theta} e^{(\theta-1)z}\left[\left(\frac{P_{T,t}}{P_t}\right)^{\theta-1}(\gamma D_t+\alpha_x MC_{T,t}H_t)\right.\\ &\quad \left. + m(1+\xi)^{1-\theta}(1+\tau)^{-\theta}q_t^{\theta}\left(\frac{P^*_{T,t}}{P^*_t}\right)^{\theta-1}\left(\gamma D^*_t+\alpha_x MC^*_{T,t}H^*_t\right)\right].\end{aligned}$$

The expected value of an entrant equals

$$V^E_{T,t} = -W_t f_E + Q_t\left(\frac{P_{t+1}}{P_t}\right)\int_{v'}\int_{z'} V_{T,t+1}(z',v',0)\,\phi_E(z')\,\phi_v(v')\,dz'dv', \qquad (70)$$

which is equal to zero in equilibrium.

The densities of productivity for exporters and non-exporters evolve as

$$\begin{aligned}\varphi_{T,t+1}(z',v',1) &= \phi_v(v')\left[\int_v\int_{z_{0,t}(v)}^{\overline{z}} n_s(z)\,\varphi_{T,t}(z,v,0)\,\phi(z'|z)\,dzdv\right. \qquad (71)\\ &\quad \left. + \int_v\int_{z_{1,t}(v)}^{\overline{z}} n_s(z)\,\varphi_{T,t}(z,v,1)\,\phi(z'|z)\,dzdv\right],\\ \varphi_{T,t+1}(z',v',0) &= \phi_v(v')\left[\int_v\int_{\underline{z}}^{z_{0,t}(v)} n_s(z)\,\varphi_{T,t}(z,v,0)\,\phi(z'|z)\,dzdv\right. \qquad (72)\\ &\quad \left. + \int_v\int_{\underline{z}}^{z_{1,t}(v)} n_s(z)\,\varphi_{T,t}(z,v,1)\,\phi(z'|z)\,dzdv + N_{TE,t}\phi_E(z')\right],\end{aligned}$$

where $N_{TE,t}$ is the mass of entrants in the tradable good sector in period t, and the masses of exporters and non-exporters in the tradable good sector are written as

$$\begin{aligned}N_{1,t} &= \int_v\int_z \varphi_{T,t}(z,v,1)\,dzdv, \qquad (73)\\ N_{0,t} &= \int_v\int_z \varphi_{T,t}(z,v,0)\,dzdv. \qquad (74)\end{aligned}$$

The total mass of producers in the tradable good sector is given as

$$N_{T,t} = N_{1,t} + N_{0,t}. \qquad (75)$$

The fixed costs of exporting imply that only a fraction $n_{x,t} = N_{1,t}/N_{T,t}$ of home tradable goods are available in the foreign country in period t.

Given the critical level of technology for exporters and non-exporters, $z_{1,t}(v)$ and $z_{0,t}(v)$, we can measure the starter ratio, the fraction of establishments that start exporting among non-exporters, as

$$n_{0,t+1} = \frac{\int_v \int_{z_{0,t(v)}}^{\overline{z}} n_s(z)\, \varphi_{T,t}(z, v, 0)\, dzdv}{\int_v \int_z n_s(z)\, \varphi_{T,t}(z, v, 0)\, dzdv}. \tag{76}$$

Similarly, we can measure the stopper ratio, the fraction of exporters who stop exporting among surviving establishments, as

$$n_{1,t+1} = \frac{\int_v \int_{\underline{z}}^{z_{1,t}(v)} n_s(z)\, \varphi_{T,t}(z, v, 1)\, dzdv}{\int_v \int_z n_s(z)\, \varphi_{T,t}(z, v, 1)\, dzdv}. \tag{77}$$

D. Government

The government collects tariffs from foreign exporters and equally distributes the tariff revenue to domestic consumers each period. The government's budget constraint is given as

$$\begin{aligned} T_t &= \tau \int_v \int_z \left[\frac{P_{F,t}(z, v, 1)}{P_t} \right] y_{F,t}(z, v, 1)\, \varphi^*_{T,t}(z, v, 1)\, dzdv \\ &= \tau (1 + \xi)^{1-\theta} (1 + \tau)^{-\theta} \left(\frac{\theta MC^*_{T,t}}{\theta - 1} \right)^{1-\theta} q_t^{1-\theta} \left(\frac{P_{T,t}}{P_t} \right)^{\theta-1} \Psi^*_{X,t} (\gamma D_t + \alpha_x MC_{T,t} H_t). \end{aligned} \tag{78}$$

E. Aggregate Variables

The total capital stock rented by intermediate good producers is given as

$$\begin{aligned} K_{t-1} &= K_{N,t} + K_{T,t} \\ &= (1 - \gamma) \left(\frac{\theta - 1}{\theta} \right) \left(\frac{\alpha}{R_t} \right) D_t + \alpha (1 - \alpha_x) \frac{MC_{T,t}}{R_t} H_t. \end{aligned} \tag{79}$$

The total labor used for production, $L_{P,t}$, is given by

$$\begin{aligned} L_{P,t} &= L_{N,t} + L_{T,t} \\ &= (1 - \gamma) \left(\frac{\theta - 1}{\theta} \right) \left(\frac{1 - \alpha}{W_t} \right) D_t + (1 - \alpha)(1 - \alpha_x) \frac{MC_{T,t}}{W_t} H_t. \end{aligned} \tag{80}$$

The total labor used by exporters for foreign market entry, $L_{X,t}$, is given by

$$L_{X,t} = f_0 \int_v \int_{z_{0,t}(v)}^{\overline{z}} e^v \varphi_{T,t}(z,v,0)\, dzdv + f_1 \int_v \int_{z_{1,t(v)}}^{\overline{z}} e^v \varphi_{T,t}(z,v,1)\, dzdv. \tag{81}$$

The total labor hired by entrants, $L_{E,t}$, is given by

$$L_{E,t} = f_E (N_{NE,t} + N_{TE,t}). \tag{82}$$

The labor market clearing condition is given by

$$L_t = L_{P,t} + L_{X,t} + L_{E,t}. \tag{83}$$

The profit of a non-tradable good producer with productivity z is given as

$$\begin{aligned} \Pi_{N,t}(z) &= \left(\frac{1-\gamma}{\theta}\right)\left(\frac{\theta MC_{N,t}}{\theta-1}\right)^{1-\theta} e^{(\theta-1)z}\left(\frac{P_{N,t}}{P_t}\right)^{\theta-1} D_t \\ &= \frac{W_t l_{N,t}(z)}{(\theta-1)(1-\alpha)}. \end{aligned} \tag{84}$$

Integrating over z, we have the total profit of non-tradable good producers as

$$\begin{aligned} \Pi_{N,t}^{all} &= \int_z \Pi_{N,t}(z)\, \varphi_{N,t}(z)\, dz \\ &= \frac{W_t L_{N,t}}{(\theta-1)(1-\alpha)}. \end{aligned} \tag{85}$$

The profit of a tradable good producer with productivity z, fixed cost, v, and exporting status m equals

$$\begin{aligned} \Pi_{T,t}(z,v,m) &= \frac{1}{\theta}\left(\frac{\theta MC_{T,t}}{\theta-1}\right)^{1-\theta} e^{(\theta-1)z}\left[\left(\frac{P_{T,t}}{P_t}\right)^{\theta-1}(\gamma D_t + \alpha_x MC_{T,t} H_t)\right. \\ &\quad \left. + m(1+\xi)^{1-\theta}(1+\tau)^{-\theta} q_t^{\theta}\left(\frac{P_{T,t}^*}{P_t^*}\right)^{\theta-1}\left(\gamma D_t^* + \alpha_x MC_{T,t}^* H_t^*\right)\right]. \end{aligned} \tag{86}$$

Integrating over z, v, and m yields total profits (excluding fixed export costs) of tradable good producers as

$$\begin{aligned}\Pi_{T,t}^{all} &= \sum_{m=0}^{1} \int_v \int_z \Pi_{N,t}(z,v,m)\, \varphi_{T,t}(z,v,m)\, dzdv \\ &= \frac{W_t L_{T,t}}{(\theta-1)(1-\alpha)(1-\alpha_x)}.\end{aligned} \tag{87}$$

As the final goods market is perfectly competitive, $\Pi_{F,t} = 0$. Thus, the aggregate profit of all producers at home country is given as

$$\begin{aligned}\Pi_t &= \Pi_{F,t} + \Pi_{N,t}^{all} + \Pi_{T,t}^{all} - W_t L_{X,t} - W_t L_{E,t} \\ &= \frac{W_t L_{N,t}}{(\theta-1)(1-\alpha)} + \frac{W_t L_{T,t}}{(\theta-1)(1-\alpha)(1-\alpha_x)} - W_t L_{X,t} - W_t L_{E,t}.\end{aligned} \tag{88}$$

The budget constraint of a home consumer is given as

$$\begin{aligned}D_t &= C_t + K_t - (1-\delta) K_t \\ &= W_t L_t + R_t K_{t-1} + \frac{W_t L_{N,t}}{(\theta-1)(1-\alpha)} + \frac{W_t L_{T,t}}{(\theta-1)(1-\alpha)(1-\alpha_x)} \\ &\quad - W_t L_{X,t} - W_t L_{E,t} + T_t \\ &= W_t L_{P,t} + R_t K_{t-1} + \frac{W_t L_{N,t}}{(\theta-1)(1-\alpha)} + \frac{W_t L_{T,t}}{(\theta-1)(1-\alpha)(1-\alpha_x)} \\ &\quad + \tau (1+\xi)^{1-\theta} (1+\tau)^{-\theta} \left(\frac{\theta MC_{T,t}^*}{\theta-1}\right)^{1-\theta} q_t^{1-\theta} \left(\frac{P_{T,t}}{P_t}\right)^{\theta-1} \Psi_{X,t}^* (\gamma D_t + \alpha_x MC_{T,t} H_t).\end{aligned} \tag{89}$$

F. Symmetric Model Solution

We consider a symmetric 2-country case. Under the symmetry, we have home variables equal to foreign variables, and the real exchange rate equals 1 always. With the normalization of prices $P_t = P_t^* = 1$, and the nominal exchange rate $e_t = 1$ under symmetry, we have the necessary equations for the model solution as follows:

From the consumer's problem and preferences of $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, we have the Euler equation given by

$$1 = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} (R_{t+1} + 1 - \delta); \tag{90}$$

and the final good price index normalization gives

$$1 = \left(\frac{P_{T,t}}{\gamma}\right)^{\gamma} \left(\frac{P_{N,t}}{1-\gamma}\right)^{1-\gamma}. \tag{91}$$

The marginal cost of a non-tradable good producer with productivity $z = 0$ is given as

$$MC_{N,t} = \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}; \tag{92}$$

the non-tradable good price index is given as

$$P_{N,t}^{1-\theta} = \left(\frac{\theta MC_{Nt}}{\theta - 1}\right)^{1-\theta} \Psi_{N,t}; \tag{93}$$

the value of a non-tradable good producer with productivity z is given as

$$\begin{aligned} V_{N,t}(z) \quad = \quad & \left(\frac{1-\gamma}{\theta}\right)\left(\frac{\theta MC_{N,t}}{\theta-1}\right)^{1-\theta} e^{(\theta-1)z} P_{N,t}^{\theta-1} D_t \\ & + n_s(z)\,\beta\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \int_{z'} V_{N,t+1}(z')\,\phi(z'|z)\,dz'; \end{aligned} \tag{94}$$

the value of an entrant in the non-tradable good sector is given as

$$V_{NE,t} = -W_t f_E + \beta\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \int_{z'} V_{N,t+1}(z')\,\phi_E(z')\,dz'; \tag{95}$$

the density of establishments in the non-tradable good sector evolves as

$$\varphi_{N,t+1}(z') = \int_z n_s(z)\,\varphi_{N,t}(z)\,\phi(z'|z)\,dz + N_{NE,t}\phi_E(z'); \tag{96}$$

and the measure of non-tradable good producers is given as

$$N_{N,t} = \int_z \varphi_{N,t}(z)\,dz. \tag{97}$$

The marginal cost of a tradable good producer with productivity $z = 0$ is given as

$$MC_{T,t} = \left(\frac{P_{T,t}}{\alpha_x}\right)^{\alpha_x} \left(\frac{MC_{N,t}}{1-\alpha_x}\right)^{1-\alpha_x}; \tag{98}$$

the price index of tradable goods is given as

$$P_{T,t}=\left[\frac{\theta}{(\theta-1)\,\alpha_x^{\alpha_x}}\left(\frac{MC_{N,t}}{1-\alpha_x}\right)^{1-\alpha_x}\right]^{\frac{1}{1-\alpha_x}}\left\{\Psi_{T,t}+\left[(1+\tau)\,(1+\xi)\right]^{1-\theta}\Psi_{X,t}\right\}^{\frac{1}{(1-\theta)(1-\alpha_x)}}; \tag{99}$$

the productivity augmented aggregate output in the tradable good sector is given as

$$H_t=\frac{\gamma\left(\frac{\theta MC_{T,t}}{\theta-1}\right)^{-\theta}\left(\frac{P_{T,t}}{P_t}\right)^{\theta-1}\left[\Psi_{T,t}+(1+\xi)^{1-\theta}(1+\tau)^{-\theta}\Psi_{X,t}\right]D_t}{1-\alpha_x\left(\frac{\theta-1}{\theta}\right)\left(\frac{\theta MC_{T,t}}{\theta-1}\right)^{1-\theta}\left(\frac{P_{T,t}}{P_t}\right)^{\theta-1}\left[\Psi_{T,t}+(1+\xi)^{1-\theta}(1+\tau)^{-\theta}\Psi_{X,t}\right]}; \tag{100}$$

the value of a tradable good producer with productivity z,fixed cost v, and exporting status m when it exports and does not are given as

$$\begin{aligned}V_{T,t}^{1}(z,v,m) &= \frac{1}{\theta}\left(\frac{\theta MC_{T,t}}{\theta-1}\right)^{1-\theta}e^{(\theta-1)z}P_{T,t}^{\theta-1}\left[1+m\,(1+\xi)^{1-\theta}(1+\tau)^{-\theta}\right] && (101)\\ &\quad(\gamma D_t+\alpha_x MC_{T,t}H_t)-W_te^{v}\left[mf_1+(1-m)\,f_0\right]\\ &\quad+n_s(z)\,\beta\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\int_{v'}\int_{z'}V_{T,t+1}\left(z',v',1\right)\phi\left(z'|z\right)\phi_v\left(v'\right)dz'dv',\\ V_{T,t}^{0}(z,v,m) &= \frac{1}{\theta}\left(\frac{\theta MC_{T,t}}{\theta-1}\right)^{1-\theta}e^{(\theta-1)z}P_{T,t}^{\theta-1}\left[1+m\,(1+\xi)^{1-\theta}(1+\tau)^{-\theta}\right] && (102)\\ &\quad(\gamma D_t+\alpha_x MC_{T,t}H_t)\\ &\quad+n_s(z)\,\beta\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\int_{v'}\int_{z'}V_{T,t+1}\left(z',v',0\right)\phi\left(z'|z\right)\phi_v\left(v'\right)dz'dv',\end{aligned}$$

respectively; the actual value of the producer is then given as

$$V_{T,t}(z,v,m)=\max\left\{V_{T,t}^{1}(z,v,m)\,,V_{T,t}^{0}(z,v,m)\right\}; \tag{103}$$

the critical values for the export decisions of exporters and non-exporters satisfy

$$\begin{aligned}z_{m,t}(v) &= \overline{z},\ \text{if } V_{T,t}^{1}(z,v,m)<V_{T,t}^{0}(z,v,m)\ \text{ for all } z\in(\underline{z},\ \overline{z})\,, && (104)\\ z_{m,t}(v) &= \underline{z},\ \text{if } V_{T,t}^{1}(z,v,m)>V_{T,t}^{0}(z,v,m)\ \text{ for all } z\in(\underline{z},\ \overline{z})\,, && (105)\\ V_{T,t}^{1}(z_{m,t}(v)\,,v,m) &= V_{T,t}^{0}(z_{m,t}(v)\,,v,m)\,,\ \text{otherwise.} && (106)\end{aligned}$$

The value of an entrant in the tradable good sector is given as

$$V_{T,t}^{E} = -W_t f_E + \beta \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \int_{v'} \int_{z'} V_{T,t+1} \left(z', v', 0 \right) \phi_E \left(z' \right) \phi_v \left(v' \right) dz' dv'; \tag{107}$$

the measure of exporters and non-exporters evolve as

$$\begin{aligned} \varphi_{T,t+1} \left(z', v', 1 \right) &= \phi_v \left(v' \right) \left[\int_v \int_{z_{0,t}(v)}^{\overline{z}} n_s \left(z \right) \varphi_{T,t} \left(z, v, 0 \right) \phi \left(z' | z \right) dz dv \right. \quad (108) \\ &\left. + \int_v \int_{z_{1,t}(v)}^{\overline{z}} n_s \left(z \right) \varphi_{T,t} \left(z, v, 1 \right) \phi \left(z' | z \right) dz dv \right], \\ \varphi_{T,t+1} \left(z', v', 0 \right) &= \phi_v \left(v' \right) \left[\int_v \int_{\underline{z}}^{z_{0,t}(v)} n_s \left(z \right) \varphi_{T,t} \left(z, v, 0 \right) \phi \left(z' | z \right) dz dv \right. \quad (109) \\ &\left. + \int_v \int_{\underline{z}}^{z_{1,t}(v)} n_s \left(z \right) \varphi_{T,t} \left(z, v, 1 \right) \phi \left(z' | z \right) dz dv + N_{TE,t} \phi_E \left(z' \right) \right], \end{aligned}$$

and the measure of exporters, non-exporters and the tradable good producers are given by

$$\begin{aligned} N_{1,t} &= \int_v \int_z \varphi_{T,t} \left(z, v, 1 \right) dz dv, \quad (110) \\ N_{0,t} &= \int_v \int_z \varphi_{T,t} \left(z, v, 0 \right) dz dv, \quad (111) \\ N_{T,t} &= N_{1,t} + N_{0,t}. \quad (112) \end{aligned}$$

The capital and labor market equilibrium condition can be written as

$$\begin{aligned} K_{t-1} &= (1 - \gamma) \left(\frac{\theta - 1}{\theta} \right) \left(\frac{\alpha}{R_t} \right) D_t + \alpha \left(1 - \alpha_x \right) \frac{MC_{T,t}}{R_t} H_t, \quad (113) \\ L_t &= (1 - \gamma) \left(\frac{\theta - 1}{\theta} \right) \left(\frac{1 - \alpha}{W_t} \right) D_t + (1 - \alpha) \left(1 - \alpha_x \right) \frac{MC_{T,t}}{W_t} H_t \quad (114) \\ &+ f_0 \int_v \int_{z_{0,t}(v)}^{\overline{z}} e^v \varphi_{T,t} \left(z, v, 0 \right) dz dv + f_1 \int_v \int_{z_{1,t}(v)}^{\overline{z}} e^v \varphi_{T,t} \left(z, v, 1 \right) dz dv \\ &+ f_E \left(N_{NE,t} + N_{NT,t} \right); \end{aligned}$$

the final goods market equilibrium condition is given as

$$D_t = C_t + K_t - (1 - \delta) K_t; \tag{115}$$

and finally the consumer's budget constraint is given as

$$
\begin{aligned}
D_t &= W_t L_{P,t} + R_t K_{t-1} + \frac{W_t L_{N,t}}{(\theta-1)(1-\alpha)} + \frac{W_t L_{T,t}}{(\theta-1)(1-\alpha)(1-\alpha_x)} \\
&\quad + \tau (1+\xi)^{1-\theta} (1+\tau)^{-\theta} \left(\frac{\theta MC_{T,t}}{\theta-1}\right)^{1-\theta} P_{T,t}^{\theta-1} \Psi_{X,t} \left(\gamma D_t + \alpha_x MC_{T,t} H_t\right).
\end{aligned} \tag{116}
$$

2. Numerical Method for Density Functions and Value Functions

We now describe the numerical method for the solution of the model.

A. Density Function Approximation

The most challenging part in solving the model is to keep track of the establishment productivity distributions, as there is not a sufficient statistics for this object. We approximate the density function from the evolution of distributions in (38), (71) and (72) as follows:

First, we choose uniformly spaced nodes for the productivity $z \in \left\{z^1, z^2, \cdots, z^J\right\}$ with an interval ω. We choose z^1 and z^J so that their absolute values are sufficiently large to not to affect the results. We approximate the transition probability and the entrants' density, $\widehat{\phi}\left(z^{j'}|z^j\right)$ and $\widehat{\phi}_E\left(z^{j'}\right)$, based on Tauchen (1986). Similarly, we choose uniformly spaced nodes for the fixed cost shock $v \in \left\{v^1, v^2, ..., v^S\right\}$ with an interval ω_v, and approximate the probability of the shock $\widehat{\phi}_v\left(v^s\right)$.[5] Then, the approximated masses of establishments evolve as

$$
\widehat{\varphi}_{N,t+1}\left(z^{j'}\right) = \sum_{j=1}^{J} n_s\left(z^j\right) \widehat{\varphi}_{N,t}\left(z^j\right) \widehat{\phi}\left(z^{j'}|z^j\right) + N_{NE,t} \widehat{\phi}_E\left(z^{j'}\right), \tag{117}
$$

$$
\begin{aligned}
\widehat{\varphi}_{T,t+1}\left(z^{j'}, v^{s'}, 1\right) &= \widehat{\phi}_v\left(v^{s'}\right) \left[\sum_{s=1}^{S} \sum_{j=1}^{J} n_s\left(z^j\right) \widehat{\varphi}_{T,t}\left(z^j, v^s, 0\right) \widehat{\phi}\left(z^{j'}|z^j\right) I_{0,t}(j,s)\right. \\
&\quad \left. + \sum_{s=1}^{S} \sum_{j=1}^{J} n_s\left(z^j\right) \widehat{\varphi}_{T,t}\left(z^j, v^s, 1\right) \widehat{\phi}\left(z^{j'}|z^j\right) I_{1,t}(j,s)\right],
\end{aligned} \tag{118}
$$

$$
\begin{aligned}
\widehat{\varphi}_{T,t+1}\left(z^{j'}, v^{s'}, 0\right) &= \widehat{\phi}_v\left(v^{s'}\right) \left[\sum_{s=1}^{S} \sum_{j=1}^{J} n_s\left(z^j\right) \widehat{\varphi}_{T,t}\left(z^j, v^s, 0\right) \widehat{\phi}\left(z^{j'}|z^j\right) \left[1 - I_{0,t}(j,s)\right]\right. \\
&\quad + \sum_{s=1}^{S} \sum_{j=1}^{J} n_s\left(z^j\right) \widehat{\varphi}_{T,t}\left(z^j, v^s, 1\right) \widehat{\phi}\left(z^{j'}|z^j\right) \left[1 - I_{1,t}(j,s)\right] \\
&\quad \left. + N_{TE,t} \widehat{\phi}_E\left(z^{j'}\right)\right],
\end{aligned} \tag{119}
$$

[5] In simulation exercises, we choose $J = 200$ and $S = 20$.

where $I_{m,t}(j,s)$ is the weight function with

$$I_{m,t}(j,s) = \begin{cases} 0 & \text{if } z^j + \omega/2 \leq z_{m,t}(v^s), \\ \frac{z^j+\omega/2-z_{m,t}(v^s)}{\omega} & \text{if } z^j - \omega/2 < z_{m,t} < z^j + \omega/2 \\ 1 & \text{if } z^j - \omega/2 \geq z_{m,t}(v^s), \end{cases}, \tag{120}$$

and $m \in \{0,\ 1\}$. This interpolation allows the approximated model to have continuity in $z_{0,t}(v^s)$ and $z_{1,t}(v^s)$.[6]

B. Value Function Approximation

Given the future values of the value functions of exporters and non-exporters, $V_{T,t+2}\left(z^j, v^s, 1\right)$ and $V_{T,t+2}\left(z^j, v^s, 0\right)$, and the current and future values of variables, we first obtain the value functions in period $t+1$ as

$$\begin{aligned} V_{T,t+1}\left(z^j, v^s, m\right) &= \Pi_{T,t+1}\left(z^j, v^s, m\right) \\ &+ \max\left\{\beta\left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\sigma} n_s\left(z^j\right)\sum_{s'=1}^{S}\sum_{j'=1}^{J} V_{T,t+2}\left(z^{j'}, v^{s'}, 0\right)\widehat{\phi}\left(z^{j'}|z^j\right)\widehat{\phi}_v\left(v^{s'}\right),\right. \\ &\beta\left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\sigma} n_s\left(z^j\right)\sum_{s'=1}^{S}\sum_{j'=1}^{J} V_{T,t+2}\left(z^{j'}, v^{s'}, 1\right)\widehat{\phi}\left(z^{j'}|z^j\right)\widehat{\phi}_v\left(v^{s'}\right) \\ &\left.-W_{t+1} f_m e^{v^s}\right\}. \end{aligned} \tag{121}$$

With these value functions, we obtain the difference of values between exporting and not exporting next period as

$$\begin{aligned} dV_{T,t}\left(z^j, v^s, m\right) &= V_{T,t}^1\left(z^j, v^s, m\right) - V_{T,t}^0\left(z^j, v^s, m\right) \\ &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} n_s\left(z^j\right)\sum_{s'=1}^{S}\sum_{j'=1}^{J}\left[V_{T,t+1}\left(z^{j'}, v^{s'}, 1\right) - V_{T,t+1}\left(z^{j'}, v^{s'}, 0\right)\right] \\ &\quad \widehat{\phi}\left(z^{j'}|z^j\right)\widehat{\phi}_v\left(v^{s'}\right) - W_t f_m e^{v^s}. \end{aligned} \tag{122}$$

[6] With the interpolation, the results become less sensitive to the choice of the number of nodes for productivity distributions compared to the complete discretization method without interpolation which creates discrete jumps in variables.

Let $z_t^{j_m}(v^s) = \max\left\{z^j | dV_{T,t}\left(z^j, v^s, m\right) < 0\right\}$ if $dV_{T,t}\left(z^j, v^s, m\right) < 0$ for some j. In this case, the thresholds for exporting, $z_{0,t}(v^s)$ and $z_{1,t}(v^s)$ are obtained as

$$z_{m,t}(v^s) = z^J - \frac{\omega dV_{T,t}\left(z^J, v^s, m\right)}{dV_{T,t}\left(z^J, v^s, m\right) - dV_{T,t}\left(z^{J-1}(v^s), v^s, m\right)}, \text{ if } j_m = J, \tag{123}$$

$$z_{m,t}(v^s) = z_t^{j_m}(v^s) - \frac{\omega dV_{T,t}\left(z_t^{j_m}(v^s), v^s, m\right)}{dV_{T,t}\left(z_t^{j_m+1}(v^s), v^s, m\right) - dV_{T,t}\left(z_t^{j_m}(v^s), v^s, m\right)}, \text{ other wise.} \tag{124}$$

If $dV_{T,t}\left(z^j, v^s, m\right) > 0$ for all j, then define

$$dV_{Tms,t} = dV_{T,t}\left(z^1, v^s, m\right) - \frac{dV_{T,t}\left(z^2, v^s, m\right) - dV_{T,t}\left(z^1, v^s, m\right)}{2}. \tag{125}$$

Then, set

$$z_{m,t}(v^s) = z^1 - \frac{\omega}{2}, \text{ if } dV_{Tms,t} > 0, \tag{126}$$

$$z_{m,t}(v^s) = z^1 - \frac{\omega dV_{T,t}\left(z^1, v^s, m\right)}{dV_{T,t}\left(z^2, v^s, m\right) - dV_{T,t}\left(z^1, v^s, m\right)}, \text{ otherwise.} \tag{127}$$

The value function of non-tradable good producer is computed as

$$V_{N,t+1}\left(z^j\right) = \Pi_{N,t+1}\left(z^j\right) + \beta\left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\sigma} n_s\left(z^j\right)\sum_{j'=1}^{J} V_{N,t+2}\left(z^{j'}\right)\widehat{\phi}\left(z^{j'}|z^j\right). \tag{128}$$

The free entry conditions in the tradable and non-tradable intermediate good sectors are

$$0 = -W_t f_E + \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\sum_{s=1}^{S}\sum_{j=1}^{J} V_{T,t+1}\left(z^j, v^s, 0\right)\widehat{\phi}_E\left(z^j\right)\widehat{\phi}_v\left(v^s\right), \tag{129}$$

$$0 = -W_t f_E + \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\sum_{j=1}^{J} V_{N,t+1}\left(z^j\right)\widehat{\phi}_E\left(z^j\right). \tag{130}$$

C. Masses of Producers and Price Indices Approximation

The masses of producers in the tradable and non-tradable good producers are obtained as

$$N_{0,t} = \sum_{s=1}^{S}\sum_{j=1}^{J}\widehat{\varphi}_{T,t}\left(z^{j},v^{s},0\right), \tag{131}$$

$$N_{1,t} = \sum_{s=1}^{S}\sum_{j=1}^{J}\widehat{\varphi}_{T,t}\left(z^{j},v^{s},1\right), \tag{132}$$

$$N_{T,t} = N_{0,t}+N_{1,t} \tag{133}$$

$$N_{N,t} = \sum_{j=1}^{J}\widehat{\varphi}_{N,t}\left(z^{j}\right). \tag{134}$$

The integration for the price indices are given as

$$\Psi_{N,t} = \sum_{j=1}^{J}e^{(\theta-1)z^{j}}\widehat{\varphi}_{N,t}\left(z^{j}\right), \tag{135}$$

$$\Psi_{X,t} = \sum_{s=1}^{S}\sum_{j=1}^{J}e^{(\theta-1)z^{j}}\widehat{\varphi}_{T,t}\left(z^{j},v^{s},1\right), \tag{136}$$

$$\Psi_{T,t} = \sum_{m=0}^{1}\sum_{s=1}^{S}\sum_{j=1}^{J}e^{(\theta-1)z^{j}}\widehat{\varphi}_{T,t}\left(z^{j},v^{s},m\right). \tag{137}$$

3. Symmetric Steady State and Calibration

We now describe how we solve for the initial and new steady states of the symmetric economy given parameter values and the key moments of the economy of interest for calibration.

A. Independently Determined Parameters

For all variations of the model, we use the following exogenously given parameters: $\beta = 0.96$, $\delta = 0.10$, $\sigma = 2$, $\theta = 5$ and $\tau = 0.08$. In the model, the tradable share of the final good producer equals $\gamma = P_T D_T/(PD)$ where the variables without time subscript denote the values of the variables in the steady state. We set γ equals to 0.21 to match the ratio of manufacturers' nominal value-added relative to private industry GDP excluding agriculture and mining for the US from 1987 to 1992. In the model, the ratio of value-added to gross output in manufacturing is given as $1-\alpha_x\left(\theta-1\right)/\theta$. The parameter for the share of materials in production, α_x, is set to match the ratio of gross output to value-added in manufacturing of 2.8. The model implied exporter's export sales to total sales ratio is given as $(1+\xi)^{1-\theta}(1+\tau)^{-\theta}/\left[1+(1+\xi)^{1-\theta}(1+\tau)^{-\theta}\right]$. We set the transportation cost parameter, ξ, to match the exporters' export sales to total sales ratio of 13.3 percent from the 1992 Census of Manufactures.

B. Jointly Determined Parameters and Initial Steady State Computation

We have 10 more parameters to set: parameters for the productivity process μ_E, ρ and σ_ε; parameter for the fixed cost shock σ_v; parameters for the death probability n_{d0} and λ, fixed and sunk cost parameters in establishment creation and export f_E, f_1, and f_0; and the capital share parameter α. We set these parameters in 2 steps.

Step 1.

In Step 1, we set the parameter values for the production technology and fixed cost shock processes together with the normalized fixed costs in exporting. We use several sub-steps in Step 1.

1. First, we normalize $N_{NE} = N_{TE} = 1$.
2. Then, we guess the first 5 parameters, μ_E, n_{d0}, ρ, σ_v, and σ_ε. Given these values, we search for the parameter values of λ, normalized f_1, and normalized f_0 to match the following key moments of establishment and exporter characteristics; (i) an exporter rate of 22.3 percent; (ii) a stopper rate of 17 percent; and (iii) a five-year exit rate of entrants of 37 percent.
   (a) In this sub-step, we obtain the normalized value functions for exporters and non-exporters and the thresholds for exporting decisions as the profit functions of these producers are proportional to $e^{(\theta-1)z}$. Let the normalized profit of exporters and non-exporters be

$$\widetilde{\Pi}\left(z^j, v^j, m\right) = \left[1 + m\left(1+\xi\right)^{1-\theta}\left(1+\tau\right)^{-\theta}\right] e^{(\theta-1)z^j}, \tag{138}$$

and the normalized fixed costs in exporting be $\widetilde{f}_m$. First, guess the values of producers, $V_N^{(i)}\left(z^j\right)$, $V_T^{(i)}\left(z^j, v^s, 0\right)$ and $V_T^{(i)}\left(z^j, v^s, 1\right)$, for all z^j and v^s. Here, the superscript (i) denotes the guessed values in the ith iteration. Then, the normalized values of producers are obtained from the iteration of the following equations:

$$\begin{aligned} \widetilde{V}_T^{(i+1)}\left(z^j, v^s, m\right) &= \left[1 + m\left(1+\xi\right)^{1-\theta}\left(1+\tau\right)^{-\theta}\right] e^{(\theta-1)z^j} \qquad (139) \\ &\quad + \max\left\{\beta n_s\left(z^j\right)\sum_{s'=1}^{S}\sum_{j'=1}^{J}\widetilde{V}_T^{(i)}\left(z^{j'}, v^{s'}, 0\right)\hat{\phi}\left(z^{j'}|z^j\right)\widehat{\phi}_v\left(v^{s'}\right),\right. \\ &\quad \left.\beta n_s\left(z^j\right)\sum_{s'=1}^{S}\sum_{j'=1}^{J}\widetilde{V}_T^{(i)}\left(z^{j'}, v^{s'}, 1\right)\hat{\phi}\left(z^{j'}|z^j\right)\widehat{\phi}_v\left(v^{s'}\right) - W\widetilde{f}_m e^{v^s}\right\}, \\ \widetilde{V}_N^{(i+1)}\left(z^j\right) &= e^{(\theta-1)z_j} + \beta n_s\left(z^j\right)\sum_{j'=1}^{J}\widetilde{V}_N^{(i)}\left(z^{j'}\right)\hat{\phi}\left(z^{j'}|z^j\right). \qquad (140) \end{aligned}$$

The expected gains from being an exporter, $\widetilde{dV}_T\left(z^j, v^s, m\right)$, for a producer with the productivity level z^j, fixed cost shock v^s, and the exporting status m equals

$$\widetilde{dV}_T\left(z^j,v^s,m\right) = \beta n_s\left(z^j\right)\sum_{s'=1}^{S}\sum_{j'=1}^{J}\left[\widetilde{V}_T\left(z^{j'},v^{s'},1\right)-\widetilde{V}_T\left(z^{j'},v^{s'},0\right)\right] \hat{\phi}\left(z^{j'}|z^j\right)\hat{\phi}_v\left(v^{s'}\right)-W\widetilde{f}_m e^{v^s}. \tag{141}$$

The critical levels of technology for exporters and non-exporters are obtained as in (123) - (127).

(b) From the evolution of measures of exporters and non-exporters in (118) and (119), we obtain the steady state measures of establishments with the normalization of $N_{NE} = N_{TE} = 1$ through the iteration of

$$\widehat{\varphi}_{T,t+1}\left(z^{j'},v^{s'},1\right) = \left[\sum_{s=1}^{S}\sum_{j=1}^{J} n_s\left(z^j\right)\widetilde{\varphi}_{T,t}\left(z^j,v^s,0\right)\widehat{\phi}\left(z^{j'}|z^j\right)I_{0,t}\left(j,s\right) + \sum_{s=1}^{S}\sum_{j=j_{1,t}}^{J} n_s\left(z^j\right)\widetilde{\varphi}_{T,t}\left(z^j,v^s,1\right)\widehat{\phi}\left(z^{j'}|z^j\right)I_{1,t}\left(j,s\right)\right]\widehat{\phi}_v\left(v^{s'}\right), \tag{142}$$

$$\widehat{\varphi}_{T,t+1}\left(z^{j'},v^{s'},0\right) = \left[\sum_{s=1}^{S}\sum_{j=1}^{J} n_s\left(z^j\right)\widetilde{\varphi}_{T,t}\left(z^j,v^s,0\right)\widehat{\phi}\left(z^{j'}|z^j\right)\left[1-I_{0,t}\left(j,s\right)\right] + \sum_{s=1}^{S}\sum_{j=1}^{J} n_s\left(z^j\right)\widetilde{\varphi}_{T,t}\left(z^j,v^s,1\right)\widehat{\phi}\left(z^{j'}|z^j\right)\left[1-I_{1,t}\left(j,s\right)\right] + N_{TE,t}\widehat{\phi}_E\left(z^{j'}\right)\right]\widehat{\phi}_v\left(v^{s'}\right), \tag{143}$$

until the densities converge. Here $I_{m,t}(j,s)$ is the weight function specified in (120).

(c) Then, we re-normalize the densities to obtain the distribution of tradable good producer productivity by multiplying the densities by the entry ratio,

$$\bar{\varphi}_T\left(z^j,v^s,1\right) = n_E\tilde{\varphi}_T\left(z^j,v^s,1\right), \tag{144}$$

$$\bar{\varphi}_T\left(z^j,v^s,0\right) = n_E\tilde{\varphi}_T\left(z^j,v^s,0\right), \tag{145}$$

where $n_E = 1/\sum_{s=1}^{S}\sum_{j=1}^{J}\left[\tilde{\varphi}_T\left(z^j,v^s,1\right)+\tilde{\varphi}_T\left(z^j,v^s,0\right)\right]$. As the productivity processes and the probabilities of death are the same across sectors, we have the non-tradable good sector distribution as

$$\bar{\varphi}_N\left(z^j\right)=\sum_{s=1}^{S}\left[\bar{\varphi}_T\left(z^j,v^s,0\right)+\bar{\varphi}_T\left(z^j,v^s,1\right)\right]. \tag{146}$$

After obtaining the distributions, we compute the steady state variables related to the distributions. The exporter rate is computed as

$$n_X=\frac{\sum_{s=1}^{S}\sum_{j=1}^{J}\bar{\varphi}_T\left(z^j,1\right)}{\sum_{s=1}^{S}\sum_{j=1}^{J}\left[\bar{\varphi}_T\left(z^j,v^s,0\right)+\bar{\varphi}_T\left(z^j,v^s,1\right)\right]}, \tag{147}$$

and the stopper rate is computed as

$$n_1=\frac{\sum_{s=1}^{S}\sum_{j=1}^{J}n_s\left(z^j\right)\bar{\varphi}_T\left(z^j,v^s,0\right)\widehat{\phi}\left(z^{j'}|z^j\right)\left[1-I_0\left(j,s\right)\right]}{\sum_{s=1}^{S}\sum_{j=1}^{J}n_s\left(z^j\right)\bar{\varphi}_T\left(z^j,v^s,1\right)}. \tag{148}$$

The normalized density of entrants is given as $\bar{\varphi}_T^{(0)}\left(z^j,v^s,0\right)=n_E\widehat{\phi}_E\left(z^j\right)\widehat{\phi}_v\left(v^s\right)$, and $\bar{\varphi}_T^{(0)}\left(z^j,v^s,1\right)=0$, where superscript (κ) denotes the normalized density of κ-year old establishments. The normalized density of κ-year old establishments is given as

$$\begin{aligned}
\overline{\varphi}_{T,t+1}^{(\kappa)}\left(z^{j'},v^{s'},1\right) &= \left[\sum_{s=1}^{S}\sum_{j=1}^{J}n_s\left(z^j\right)\overline{\varphi}_{T,t}^{(\kappa-1)}\left(z^j,v^s,0\right)\widehat{\phi}\left(z^{j'}|z^j\right)I_{0,t}\left(j,s\right)\right. && (149)\\
&\quad\left.+\sum_{s=1}^{S}\sum_{j=j_{1,t}}^{J}n_s\left(z^j\right)\overline{\varphi}_{T,t}^{(\kappa-1)}\left(z^j,v^s,1\right)\widehat{\phi}\left(z^{j'}|z^j\right)I_{1,t}\left(j,s\right)\right]\widehat{\phi}_v\left(v^{s'}\right),\\
\overline{\varphi}_{T,t+1}^{(\kappa)}\left(z^{j'},v^{s'},0\right) &= \left[\sum_{s=1}^{S}\sum_{j=1}^{J}n_s\left(z^j\right)\overline{\varphi}_{T,t}^{(\kappa-1)}\left(z^j,v^s,0\right)\widehat{\phi}\left(z^{j'}|z^j\right)\left[1-I_{0,t}\left(j,s\right)\right]\right. && (150)\\
&\quad\left.+\sum_{s=1}^{S}\sum_{j=1}^{J}n_s\left(z^j\right)\overline{\varphi}_{T,t}^{(\kappa-1)}\left(z^j,v^s,1\right)\widehat{\phi}\left(z^{j'}|z^j\right)\left[1-I_{1,t}\left(j,s\right)\right]\right]\widehat{\phi}_v\left(v^{s'}\right).
\end{aligned}$$

We obtain the 5-year exit rate as

$$1-\frac{\sum_{m=0}^{1}\sum_{s=1}^{S}\sum_{j=1}^{J}\bar{\varphi}_T^{(5)}\left(z^{j'},v^s,m\right)}{\sum_{s=1}^{S}\sum_{j=1}^{J}\bar{\varphi}_T^{(0)}\left(z^j,v^s,0\right)}. \tag{151}$$

3. With sub-steps $(2.a)-(2.c)$, we search for the values of μ_E, n_{d0}, ρ, and σ_v, for a given value of σ_ε to match entrants' labor share of 1.5 percent and the shutdown establishments' labor share of 2.3 percent, and to minimize the sum of squared residuals between the data and the model's implied distributions of establishments and export participation rates in the 1992 Census of Manufactures. For the distributions, we use 10 bins for the employment size distribution: establishments with $1-4$, $5-9$, $10-19$, $20-49$, $50-99$, $100-249$, $250-499$, $500-999$, $1,000-2,499$, and $2,500+$ employees and 6 bins for the export participation rate distribution: establishments with $1-99$, $100-249$, $250-499$, $500-999$, $1000-2499$, and $2500+$ employees for the export participation rates that are available in the 1992 Census of Manufacturers.

   (a) From (59) and (63), we have the employment level of an establishment proportional to the productivity

   $$l_T(z,m) \propto \left[1+m(1+\xi)^{1-\theta}(1+\tau)^{-\theta}\right]e^{(\theta-1)z}, \tag{152}$$

   where $\propto$ denotes 'proportional to'. The normalized aggregate level of labor in production in the tradable good sector is given as

   $$\bar{L}_T = \sum_{s=1}^{S}\sum_{j=1}^{J} e^{(\theta-1)z^j}\left\{\bar{\varphi}_T\left(z^j,v^s,0\right)+\left[1+m(1+\xi)^{1-\theta}(1+\tau)^{-\theta}\right]\bar{\varphi}_T\left(z^j,v^s,1\right)\right\}, \tag{153}$$

   the shut-down establishments' labor share is given as

   $$\begin{aligned}\bar{l}_{TD} &= \frac{1}{\bar{L}_T}\sum_{s=1}^{S}\sum_{j=1}^{J}\left[1-n_s\left(z^j\right)\right]e^{(\theta-1)z^j}\left\{\bar{\varphi}_T\left(z^j,v^s,0\right)\right. \\ &\quad \left.+\left[1+m(1+\xi)^{1-\theta}(1+\tau)^{-\theta}\right]\bar{\varphi}_T\left(z^j,v^s,1\right)\right\},\end{aligned} \tag{154}$$

   and the new establishments' labor share is given as

   $$\bar{l}_{TE} = \frac{\sum_{j=1}^{J} e^{(\theta-1)z^j} n_E \widehat{\phi}_E\left(z^j\right)}{\bar{L}_T}. \tag{155}$$

   (b) For the computation of the establishment size distribution, we rescale the employment levels with the average employment level of establishments in the U.S. manufacturing sector in 1992. The employment level for an establishment with productivity z, fixed cost shock v, and exporting status m is given as

   $$\tilde{l}(z,v,m) = \frac{e^{(\theta-1)z^j}\left[1+m(1+\xi)^{1-\theta}(1+\tau)^{-\theta}\right]}{\bar{L}_T}\frac{L_{1992}}{N_{1992}}, \tag{156}$$

where L_{1992} and N_{1992} are the total employment level and the total number of establishments in the U.S. manufacturing sector in 1992. The cumulative mass of establishments who employ less than or equal to l units is given as

$$G_e(l) = \sum_{s=1}^{S}\sum_{j=1}^{J}\left\{\bar{\varphi}_T\left(z^j, v^s, 0\right) I\left[\tilde{l}\left(z^j, v^s, 0\right) \leq l\right] + \bar{\varphi}_T\left(z^j, v^s, 1\right) I\left[\tilde{l}\left(z^j, v^s, 1\right) \leq l\right]\right\}, \tag{157}$$

where $I\left[\tilde{l}\left(z^j, v^s, 0\right) \leq l\right]$ is an indicator function, $I\left[\tilde{l}\left(z^j, v^s, 0\right) \leq l\right]$ equals 1 if $\tilde{l}\left(z^j, v, 0\right) \leq l$ is true and 0 otherwise. The cumulative mass of employment level for the establishment size less than equal to l units is given as

$$\begin{aligned} G_l(l) \;=\; & \sum_{s=1}^{S}\sum_{j=1}^{J}\left\{\tilde{l}\left(z^j, v^s, 0\right)\bar{\varphi}_T\left(z^j, v^s, 0\right) I\left[\tilde{l}\left(z^j, v^s, 0\right) \leq l\right]\right. \\ & \left. +\tilde{l}\left(z^j, v^s, 1\right)\bar{\varphi}_T\left(z^j, v^s, 1\right) I\left[\tilde{l}\left(z^j, v^s, 1\right) \leq l\right]\right\}\frac{N_{1992}}{L_{1992}}. \end{aligned} \tag{158}$$

The cumulative mass of exporters and non-exporters are given as

$$\begin{aligned} G_1(l) &= \sum_{s=1}^{S}\sum_{j=1}^{J}\bar{\varphi}_T\left(z^j, v^s, 1\right) I\left[\tilde{l}\left(z^j, v^s, 1\right) \leq l\right], \\ G_0(l) &= \sum_{s=1}^{S}\sum_{j=1}^{J}\bar{\varphi}_T\left(z^j, v^s, 0\right) I\left[\tilde{l}\left(z^j, v^s, 0\right) \leq l\right]. \end{aligned}$$

The probability mass functions of establishments, employments, and export participation rates for the employment levels between l_0 and l_1 are given by

$$g_e(l_0, l_1) = G_e(l_1) - G_e(l_0), \tag{159}$$

$$g_l(l_0, l_1) = G_l(l_1) - G_l(l_0), \tag{160}$$

$$g_x(l_0, l_1) = \frac{G_1(l_1) - G_1(l_0)}{G_e(l_1) - G_e(l_0)}. \tag{161}$$

(c) With the entrants' labor share of 1.5 percent and the shutdown establishments' labor share of 2.3 percent being matched, we search for a set of parameter values which minimizes the sum of squared residuals between the data and the model's implied distributions

$$\min\left\{\sum_{a=1}^{10}\left[g_e\left(l_a,l_{a+1}\right)_{\text{model}}-g_e\left(l_a,l_{a+1}\right)_{\text{data}}\right]^2,\right. \tag{162}$$
$$\left.+\sum_{b=1}^{6}\left[g_x\left(l^b,l^{b+1}\right)_{\text{model}}-g_x\left(l^b,l^{b+1}\right)_{\text{data}}\right]^2\right\}$$

where l_a and l^b are the lower bounds of an employment bin specified in the data.

4. With sub-steps $1-3$, we search for the value of σ_ε to minimize the sum of squared residuals from the data and the model's implied distributions of establishments and export participation rates in the 1992 Census of Manufactures.

Step 2.

In Step 2, we obtain the steady state values of variables together with the entry and exporting fixed cost parameter values with the normalization of labor supply, $L=1$, and the normalization of the measure of establishments, $N_N+N_T=2$.

1. First, guess the steady state values of K, N_T, N_N, C and W, along with the remaining parameter values to be set, f_E and α. Given the guessed values, the steady state total sales of final good producers from (115) equal
$$D=C+\delta K. \tag{163}$$

   The marginal cost of a non-tradable good producer with $z=0$, MC_N, is obtained from (92) as

$$MC_N=\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}, \tag{164}$$

   where R is given by the steady state representation of (90), $R=1/\beta-1+\delta$. Since the tradable and non-tradable good sectors have the same productivity process and death probability, the price index of non-tradable goods is obtained from (93) as

$$P_N=\left(\frac{\theta}{\theta-1}\right)MC_N\left(N_N\bar{\Psi}_N\right)^{\frac{1}{1-\theta}}, \tag{165}$$

   where $\bar{\Psi}_N=\sum_{s=1}^{S}\sum_{j=1}^{J}e^{(\theta-1)z^j}\left[\bar{\varphi}_T\left(z^j,v^s,0\right)+\bar{\varphi}_T\left(z^j,v^s,1\right)\right]$. The price index of tradable goods is obtained from (99) as

$$P_T=\left[\frac{\theta}{(\theta-1)\,\alpha_x^{\alpha_x}}\right]^{\frac{1}{1-\alpha_x}}\left(\frac{MC_N}{1-\alpha}\right)\left[N_T\bar{\Psi}_T+(1+\tau)^{1-\theta}(1+\iota)^{1-\theta}N_T\bar{\Psi}_X\right]^{\frac{1}{(1-\theta)(1-\alpha_x)}}, \tag{166}$$

where $\bar{\Psi}_T = \bar{\Psi}_N$ and $\bar{\Psi}_X = \sum_{s=1}^{S} \sum_{j=1}^{J} e^{(\theta-1)z^j} \bar{\varphi}_T \left(z^j, v^s, 1\right)$. Then, the marginal cost of a tradable good producer with $z = 0$ is obtained from (98) as

$$MC_T = \left(\frac{P_T}{\alpha_x}\right)^{\alpha_x} \left(\frac{MC_N}{1-\alpha_x}\right)^{1-\alpha_x}. \tag{167}$$

The productivity adjusted aggregate output, H, is obtained from (100) as

$$H = \frac{\gamma \left(\frac{\theta MC_T}{\theta-1}\right)^{-\theta} P_T^{\theta-1} N_T \left[\bar{\Psi}_T + (1+\xi)^{1-\theta} (1+\tau)^{-\theta} \bar{\Psi}_{X,t}\right] D}{1 - \alpha_x \left(\frac{\theta-1}{\theta}\right) \left(\frac{\theta MC_T}{\theta-1}\right)^{1-\theta} P_T^{\theta-1} N_T \left[\bar{\Psi}_T + (1+\xi)^{1-\theta} (1+\tau)^{-\theta} \bar{\Psi}_X\right]}. \tag{168}$$

2. Given the values, we obtain the fixed costs in exporting and the values of producers from the following sub-procedures:
   (a) First, define

$$\Gamma_T = \left(\frac{1}{\theta}\right) \left\{ \frac{\gamma D + \alpha_x MC_T H}{N_T \left[\bar{\Psi}_T + (1+\xi)^{1-\theta} (1+\tau)^{1-\theta} \bar{\Psi}_{X,t}\right]} \right\}, \tag{169}$$

$$\Gamma_N = \left(\frac{1-\gamma}{\theta}\right) e^{(\theta-1)z_j} \left(\frac{D}{N_N \bar{\Psi}_N}\right). \tag{170}$$

   (b) Then, the fixed costs in exporting are obtained as

$$f_m = \frac{\widetilde{f}_m \Gamma_T}{W}, \tag{171}$$

   and the values of firms as

$$\widehat{V}_T \left(z^j, v^s, m\right) = \Gamma_T \widetilde{V} \left(z^j, v^s, m\right), \tag{172}$$

$$\widehat{V}_N \left(z^j\right) = \Gamma_N \widetilde{V} \left(z^j\right). \tag{173}$$

3. Total exports are obtained from (56) and the equilibrium condition $y_H^* \left(z^j, v^s, 1\right) = y_H^{d*} \left(z^j, v^s, 1\right) + \sum_{\mu=0}^{1} \sum_{s^*=1}^{S} \sum_{j^*=1}^{J} x_H^{d*}(z^j, v^s, 1, \zeta^{j^*}, \varpi^{s^*}, \mu) \varphi_{T,t}^* \left(\zeta^{j^*}, \varpi^{s^*}, \mu\right)$ with (46), (60), the foreign analog

of (42), and (166),

$$\begin{aligned}
EX &= N_T \sum_{s=1}^{S}\sum_{j=1}^{J} P_H^{*}\left(z^j, v^s, 1\right) y_H^{*}\left(z^j, v^s, 1\right) \bar{\varphi}_T\left(z^j, v^s, 1\right) \\
&= \left(1+\tau\right)^{-\theta}\left(1+\xi\right)^{1-\theta}\left(\frac{\theta MC_T}{\theta-1}\right)^{1-\theta} P_T^{\theta-1} N_T \bar{\Psi}_X\left(\gamma D+\alpha_x MC_T H\right) \\
&= \left(\gamma D+\alpha_x MC_T H\right)\left[\frac{\left(1+\tau\right)^{-\theta}\left(1+\iota\right)^{1-\theta}\bar{\Psi}_X}{\bar{\Psi}_T+\left(1+\tau\right)^{1-\theta}\left(1+\iota\right)^{1-\theta}\bar{\Psi}_X}\right],
\end{aligned} \tag{174}$$

and the aggregate capital stocks used in the tradable and non-tradable good sectors are obtained from (**??**) and (68) as

$$K_N = (1-\gamma)\left(\frac{\theta-1}{\theta}\right)\left(\frac{\alpha}{R}\right)D, \tag{175}$$

$$K_T = (1-\alpha_x)\left(\frac{\alpha}{R}\right)MC_T H. \tag{176}$$

Labor used in the tradable and non-tradable good sectors are obtained from (35) and (67) as

$$L_T = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R}{W}\right)K_T, \tag{177}$$

$$L_N = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R}{W}\right)K_N. \tag{178}$$

We obtain the labor in production from (80) as

$$L_p = L - f_E(N_{TE}+N_{NE}) - f_0 N_T n_{01} - f_1 N_T n_{11}, \tag{179}$$

where $N_{TE} = n_E N_T$ and $N_{NE} = n_E N_{N,}$ and n_{01} and n_{11} are the normalized mass of non-exporters who decide to enter the foreign market and the normalized mass of exporters who decide to stay in the foreign market,

$$n_{01} = \sum_{s=1}^{S}\sum_{j=1}^{J}\bar{\varphi}_T\left(z^j, v^s, 0\right) I_0\left(j, s\right), \tag{180}$$

$$n_{11} = \sum_{s=1}^{S}\sum_{j=1}^{J}\bar{\varphi}_T\left(z^j, v^s, 1\right) I_1\left(j, s\right). \tag{181}$$

4. Using the following 7 equations, we update the initial guesses of variables and parameters:
   (a) the normalization of mass of producers,

$$N_N + N_T = 2, \tag{182}$$

   (b) the normalization of the final good price in (19),

$$1 = \left(\frac{P_T}{\gamma}\right)^{\gamma}\left(\frac{P_N}{1-\gamma}\right)^{1-\gamma}, \tag{183}$$

   (c) the free entry conditions in the tradable and non-tradable good sectors in (129) and (130),

$$W f_E = \beta \sum_{s=1}^{S}\sum_{j=1}^{J} V_T\left(z^j, v^s, 0\right)\hat{\phi}_E\left(z^j\right)\hat{\phi}_v\left(v^s\right), \tag{184}$$

$$W f_E = \beta \sum_{j=1}^{J} V_N\left(z^j\right)\hat{\phi}_E\left(z^j\right), \tag{185}$$

   (d) the labor income share of 66 percent,

$$\frac{WL}{D} = 0.66, \tag{186}$$

   (e) the capital market clearing condition,

$$K = K_T + K_N, \tag{187}$$

   (f) the budget constraint of the representative consumer in (116),

$$D = WL_P + RK + \left[\frac{W}{(1-\alpha)(1-\theta)}\right]\left[L_N + \left(\frac{1}{1-\alpha_x}\right)L_T\right] + \tau EX. \tag{188}$$

5. Repeat all the procedures until all the guessed values converges.

C. New Steady State Computation

In the computation of a new steady state, we maintain the parameter values obtained in the initial steady state. With a new tariff rate, the values of variables change and the critical levels of technology for exporters and non-exporters change, and thus, the distributions of establishments change as well. We use the following steps to compute the values in a new steady state

1. Guess the values of K, C, W, N_T, N_N, the exporting thresholds, and the productivity distributions. Then, update the guessed values of K, C, W, N_T, and N_N using the following 5 equations together with values of producers:

   (a) the normalization of the final good price,

$$1 = \left(\frac{P_T}{\gamma}\right)^{\gamma}\left(\frac{P_N}{1-\gamma}\right)^{1-\gamma}, \tag{189}$$

   (b) the free entry conditions in the tradable and non-tradable good sectors,

$$W f_E = \beta \sum_{s=1}^{S}\sum_{j=1}^{J} V_T\left(z^j, v^s, 0\right)\hat{\phi}_E\left(z^j\right)\hat{\phi}_v\left(v^s\right), \tag{190}$$

$$W f_E = \beta \sum_{j=1}^{J} V_N\left(z^j\right)\hat{\phi}_E\left(z^j\right), \tag{191}$$

   (c) the capital market clearing condition,

$$K = K_T + K_N, \tag{192}$$

   (d) the budget constraint of the representative consumer,

$$D = WL_P + RK + \left[\frac{W}{(1-\alpha)(1-\theta)}\right]\left[L_N + \left(\frac{1}{1-\alpha_x}\right)L_T\right] + \tau EX. \tag{193}$$

   Here, we update the value functions through the iterations of

$$\begin{aligned}\widehat{V}_T\left(z^j, v^s, m\right) &= \Gamma_T\left[1 + m\left(1+\xi\right)^{1-\theta}\left(1+\tau\right)^{-\theta}\right]e^{(\theta-1)z^j} \\ &+ \max\left\{\beta n_s\left(z^j\right)\sum_{s'=1}^{S}\sum_{j'=1}^{J}\widehat{V}_T\left(z^{j'}, v^{s'}, 0\right)\hat{\phi}\left(z^{j'}|z^j\right)\widehat{\phi}_v\left(v^{s'}\right),\right. \\ &\left.\beta n_s\left(z^j\right)\sum_{s'=1}^{S}\sum_{j'=1}^{J}\widehat{V}_T\left(z^{j'}, v^{s'}, 1\right)\hat{\phi}\left(z^{j'}|z^j\right)\widehat{\phi}_v\left(v^{s'}\right) - W f_m e^{v^s}\right\},\end{aligned} \tag{194}$$

$$\widehat{V}_N\left(z^j\right) = \Gamma_N e^{(\theta-1)z_j} + \beta n_s\left(z^j\right)\sum_{j'=1}^{J}\widehat{V}_N\left(z^{j'}\right)\hat{\phi}\left(z^{j'}|z^j\right). \tag{195}$$

2. With the updated value functions, update the exporting thresholds as described in the initial steady state computations, and update the distributions of exporters and non-exporters described in Step 1.
3. Repeat steps 1 and 2 until the guessed variables converge.

4. Procedure for Computing Transition Dynamics

The following describes the procedure for the computation of transition dynamics.

Step 1.

In the first step, we first solve for the steady states before and after the trade liberalization. After getting the steady state distribution, we work with the measures of establishments, $\widehat{\varphi}_{T,t}\left(z^j, v^s, 1\right)$, $\widehat{\varphi}_{T,t}\left(z^j, v^s, 0\right)$, and $\widehat{\varphi}_{N,t}\left(z^j\right)$, rather than the normalized measures, $\bar{\varphi}_{T,t}\left(z^j, v^s, 1\right)$, $\bar{\varphi}_{T,t}\left(z^j, v^s, 0\right)$, and $\bar{\varphi}_{N,t}\left(z^j\right)$, to deal with the time varying measures of establishments. We then assume that at $t = 0$ the initial steady state is maintained, and at $t = 1$ the tariff rate is eliminated unexpectedly. We further assume that the new steady state is achieved in T periods, where we set T sufficiently large so that the resulting transitions are extremely insensitive to an increase in T.[7] We set the initial guesses of the sequences of key variables that are used to obtain the equilibrium for the transition dynamics, $\{C_t^{(0)}, W_t^{(0)}, R_t^{(0)}, \left\{z_{0,t}^{(0)}\left(v^s\right), z_{1,t}^{(0)}\left(v^s\right)\right\}_{s=1}^{S}, N_{TE,t}^{(0)}, N_{NE,t}^{(0)}, K_t^{(0)}\}_{t=1}^{T}$, and $\{\widehat{\varphi}_{T,t}^{(0)}\left(z^j, v^s, 1\right), \widehat{\varphi}_{T,t}^{(0)}\left(z^j, v^s, 0\right), \widehat{\varphi}_{N,t}^{(0)}\left(z^j\right), V_{T,t+1}^{(0)}\left(z^j, v^s, 1\right), V_{T,t+1}^{(0)}\left(z^j, v^s, 0\right), V_{N,t+1}^{(0)}\left(z^j\right)\}_{t=1}^{T}{}_{s=1}^{S}{}_{j=1}^{J}$. Here, superscript (0) denotes the initially guessed values.[8]

Step 2.

We use several sub-steps in the second step as follows.

1. Guess the values of exporting decision thresholds, $\left\{z_{0,t}^{(i)}\left(v^s\right),\ z_{1,t}^{(i)}\left(v^s\right)\right\}_{s=1}^{S}$ Here superscript (i) denotes the values guessed at i_{th} iteration.
2. Guess the values of $N_{TE,t}^{(i)}$, $N_{NE,t}^{(i)}$, $K_t^{(i)}$, $C_t^{(i)}$, $W_t^{(i)}$, $N_{TE,t+1}^{(i)}$, $N_{NE,t+1}^{(i)}$, $C_{t+1}^{(i)}$, and $W_{t+1}^{(i)}$.[9]
3. Update the masses of establishments:
   (a) compute $\{\widehat{\varphi}_{T,t+1}^{(i)}\left(z^j, v^s, 1\right), \widehat{\varphi}_{T,t+1}^{(i)}\left(z^j, v^s, 0\right), \widehat{\varphi}_{N,t+1}^{(i)}\left(z^j\right)\}{}_{s=1}^{S}{}_{j=1}^{J}$ with $\left\{z_{0,t}^{(i)}\left(v^s\right),\ z_{1,t}^{(i)}\left(v^s\right)\right\}_{s=1}^{S}$, $N_{TE,t}^{(i)}$, and $N_{NE,t}^{(i)}$;
   (b) compute $\{\widehat{\varphi}_{T,t+2}^{(i)}\left(z^j, v^s, 1\right), \widehat{\varphi}_{T,t+2}^{(i)}\left(z^j, v^s, 0\right), \widehat{\varphi}_{N,t+2}^{(i)}\left(z^j\right)\}{}_{s=1}^{S}{}_{j=1}^{J}$ with $\left\{z_{0,t+1}^{(i-1)}\left(v^s\right),\ z_{1,t+1}^{(i-1)}\left(v^s\right)\right\}_{s=1}^{S}$, $N_{TE,t+1}^{(i)}$, and $N_{NE,t+1}^{(i)}$; and
   (c) compute $\left\{\Psi_{N,,t+j}^{(i)},\ \Psi_{T,t+j}^{(i)},\ \Psi_{X,t+j}^{(i)}\right\}_{j=0}^{2}$, and $\left\{N_{N,t+j}^{(i)},\ N_{0,t+j}^{(i)},\ N_{1,t+j}^{(i)},\ N_{T,t+j}^{(i)}\right\}_{j=0}^{1}$, and $\{N_{01,t+j},\ N_{11,t+j}\}_{j=0}^{1}$.

[7] In the simulations, we set $T = 300$. The results show that all the variables become very close to the new steady state in $t = 100$.

[8] The new steady state values can be good candidates for the initial guesses if the tariff cut is not too drastic.

[9] Here, we use not only current variables but also some 1 period ahead variables to allow for greater flexibility as the exporting decisions are based on expected values of producers in the next period.

4. Given the guesses and the updated distributions, obtain values of other key variables for the solution:
   (a) Obtain $\left\{L_{P,t+j}^{(i)},\ R_{t+j}^{(i)},\ MC_{N,t+j}^{(i)},\ P_{N,t+j}^{(i)},\ P_{T,t+j}^{(i)},\ MC_{T,t+j}^{(i)},\ H_{t+j}^{(i)}\right\}_{j=0}^{1}$.
   (b) Given the guessed values of producers in period $t+2$, and the values of variables computed, obtain the one period ahead value functions of non-tradable and tradable good producers, $\left\{V_{T,t+1}^{(i)}\left(z^j, v^s, m\right),\ V_{N,t+1}^{(i)}\left(z^j\right)\right\}\ {}_{s=1}^{S}\ {}_{j=1}^{J}$ from

$$
\begin{aligned}
V_{T,t+1}^{(i)}\left(z^j, v^s, m\right) &= \Pi_{T,t+1}^{(i)}\left(z^j, v^s, m\right) \\
&+ \max\left\{\beta\left(\frac{C_{t+2}^{(i-1)}}{C_{t+1}^{(i)}}\right)^{-\sigma} n_s\left(z^j\right)\sum_{s'=1}^{S}\sum_{j'=1}^{J} V_{T,t+2}^{(i-1)}\left(z^{j'}, v^{s\prime}, 0\right)\right. \\
&\widehat{\phi}\left(z^{j'}|z^j\right)\widehat{\phi}_v\left(v^{s'}\right), \\
&-W_{t+1}^{(i)} f_m e^v + \beta\left(\frac{C_{t+2}^{(i-1)}}{C_{t+1}^{(i)}}\right)^{-\sigma} n_s\left(z^j\right)\sum_{j'=1}^{n} V_{T,t+2}^{(i-1)}\left(z^{j'}, v^{s\prime}, 1\right) \\
&\left.\widehat{\phi}\left(z^{j'}|z^j\right)\widehat{\phi}_v\left(v^{s'}\right)\right\},
\end{aligned} \tag{196}
$$

$$
\begin{aligned}
V_{N,t+1}^{(i)}\left(z^j\right) &= \Pi_{N,t+1}^{(i)}\left(z^j\right) \\
&+\beta\left(\frac{C_{t+2}^{(i-1)}}{C_{t+1}^{(i)}}\right)^{-\sigma} n_s\left(z^j\right)\sum_{j'=1}^{n} V_{N,t+2}^{(i-1)}\left(z^{j'}\right)\widehat{\phi}\left(z^{j'}|z^j\right),
\end{aligned} \tag{197}
$$

where

$$
\begin{aligned}
\Pi_{T,t+1}^{(i)}\left(z^j, v^s, m\right) &= \frac{1}{\theta}\left(\frac{\theta MC_{T,t+1}^{(i)}}{\theta-1}\right)^{1-\theta} e^{(\theta-1)z^j} P_{T,t+1}^{(i)\theta-1} \\
&\left[1+m\left(1+\xi\right)^{1-\theta}\left(1+\tau\right)^{-\theta}\right] \\
&\left(\gamma D_{t+1}^{(i)} + \alpha_x MC_{T,t+1}^{(i)} H_{t+1}^{(i)}\right),
\end{aligned} \tag{198}
$$

$$
\Pi_{N,t+1}^{(i)}\left(z^j\right) = \left(\frac{1-\gamma}{\theta}\right)\left(\frac{\theta MC_{N,t+1}^{(i)}}{\theta-1}\right)^{1-\theta} e^{(\theta-1)z^j} P_{N,t+1}^{(i)\theta-1} D_{t+1}^{(i)}, \tag{199}
$$

$$
D_{t+1}^{(i)} = C_{t+1}^{(i)} + K_{t+1}^{(i-1)} - (1-\delta)K_t^{(i)}.
$$

5. Evaluate the guessed values using the following equations:

the Euler equation of consumers in (90),

$$1 = \beta \left(\frac{C_{t+1}^{(i)}}{C_t^{(i)}} \right)^{-\sigma} \left(R_{t+1}^{(i)} + 1 - \delta \right); \tag{200}$$

the free entry conditions in the tradable and non-tradable good sectors in period t,

$$W_t^{(i)} f_E = \beta \left(\frac{C_{t+1}^{(i)}}{C_t^{(i)}} \right)^{-\sigma} \sum_{s=1}^{S} \sum_{j=1}^{J} V_{T,t+1}^{(i)} \left(z^j, v^s, 0 \right) \hat{\phi}_E \left(z^j \right) \hat{\phi}_v \left(v^s \right), \tag{201}$$

$$W_t^{(i)} f_E = \beta \left(\frac{C_{t+1}^{(i)}}{C_t^{(i)}} \right)^{-\sigma} \sum_{j=1}^{J} V_{N,t+1}^{(i)} \left(z^j \right) \hat{\phi}_E \left(z^j \right), \tag{202}$$

and in period $t+1$,

$$W_{t+1}^{(i)} f_E = \beta \left(\frac{C_{t+2}^{(i-1)}}{C_{t+1}^{(i)}} \right)^{-\sigma} \sum_{s=1}^{S} \sum_{j=1}^{J} V_{T,t+2}^{(i-1)} \left(z^j, v^s, 0 \right) \hat{\phi}_E \left(z^j \right) \hat{\phi}_v \left(v^s \right), \tag{203}$$

$$W_{t+1}^{(i)} f_E = \beta \left(\frac{C_{t+2}^{(i-1)}}{C_t^{(i)}} \right)^{-\sigma} \sum_{j=1}^{J} V_{N,t+2}^{(i-1)} \left(z^j \right) \hat{\phi}_E \left(z^j \right); \tag{204}$$

the final goods price normalization in period t,

$$1 = \left(\frac{P_{T,t}^{(i)}}{\gamma} \right)^{\gamma} \left(\frac{P_{N,t}^{(i)}}{1-\gamma} \right)^{1-\gamma}, \tag{205}$$

and in period $t+1$

$$1 = \left(\frac{P_{T,t+1}^{(i)}}{\gamma} \right)^{\gamma} \left(\frac{P_{N,t+1}^{(i)}}{1-\gamma} \right)^{1-\gamma}; \tag{206}$$

and the labor market equilibrium conditions (80) in period t,

$$L_{P,t}^{(i)} = (1-\gamma) \left(\frac{\theta - 1}{\theta} \right) \left(\frac{1-\alpha}{W_t^{(i)}} \right) D_t^{(i)} + (1-\alpha)(1-\alpha_x) \frac{MC_{T,t}^{(i)}}{W_t^{(i)}} H_t^{(i)}, \tag{207}$$

and in period $t+1$,

$$L_{P,t+1}^{(i)} = (1-\gamma)\left(\frac{\theta-1}{\theta}\right)\left(\frac{1-\alpha}{W_{t+1}^{(i)}}\right)D_{t+1}^{(i)} + (1-\alpha)(1-\alpha_x)\frac{MC_{T,t+1}^{(i)}}{W_{t+1}^{(i)}}H_{t+1}^{(i)}. \tag{208}$$

As the entry decisions are made 1 period ahead of actual production, the masses of entrants in the tradable and non-tradable good sectors have periodic oscillation if the entry costs are constant. To reduce this oscillating behavior of masses of entrants, we impose small time varying entry cost which is invariant in any steady states. Specifically, we use the entry cost of $f_{Ei,t} = f_E\left[\frac{(\kappa-1)N_{iE,t}}{\kappa n_E N_{i,t} - N_{iE,t}}\right]^{\psi}$, $i \in \{T, N\}$ with $\kappa = 10$ and $\psi = 0.2$.[10]

6. Repeat steps $2-5$ until the all the equations are satisfied.
7. Given the updated values from steps $1-6$, update the exporting decision thresholds, $\left\{z_{0,t}^{(i)}(v^s),\ z_{1,t}^{(i)}(v^s)\right\}_{s=1}^{S}$ using the updated values of producers.
8. Repeat steps $1-7$ until all the guessed values converge.
9. Do the procedure of steps 1 through 8 for $t=1$ through $t=T$.
10. Repeat steps 1 through 9 until all the sequences of variables converge.

5. Numerical Precision

In this section, we evaluate the precision of our numerical approximation in a simplified model in which we can obtain an exact analytical solution for the long run growth and transitions. We compare the solution from our numerical method to the analytical solution. We find our numerical methods accurately capture both the change in the steady state and the transition path following a cut in trade costs. Specifically, for this variation our approximation of the change in steady state consumption is within 0.15 percent (1.0541 percent vs. 1.0524 percent). Along the transition the gap is also quite small with the difference in cumulative consumption over the first 50 periods of 0.099 percent and a root mean square error along the path of only 0.001 percent. Similar scale errors in our full-blown model would suggest our findings are accurate to 3 or 4 digits.

We make the following restrictions in this section:

1. Fixed cost in exporting: $f_0 = f_1$.
2. No entrants' disadvantage: constant n_s and $\mu_E = 0$.
3. Pareto distribution of productivity: $a = e^{(\theta-1)z}$ follows a Pareto distribution with $\phi(a) = \eta a^{-1-\eta}$. Note that with no entrants' disadvantage together with the fixed cost structure, persistence of a does not affect the results.
4. Only iceberg costs: we eliminate tariffs, $\tau = 0$, and consider a cut in the iceberg cost, ξ.
5. Zero time discount rate: we consider the limit case where $\beta \to 1$.

With these assumptions, we will evaluate the performance of the approximation method by

[10] One can obtain the transition dynamics with constant entry costs by reducing the value of ψ toward 0 through iterations.

using the approximation method for the distributions, the value functions of producers, and the export threshold even though we can obtain the growth rates of key variables with the exact distributions, value functions, and the export threshold. We then compare the results with the exact numerical solutions. Additionally, we will check the performance of the log-linearization method with the system of equations for the long-run equilibrium. We will consider 3 cases: i) a model with material inputs and capital, ii) a model without material inputs, and iii) a model only with labor inputs.

Parameter values All the parameter values are the same as the benchmark case except the following:

1. $\alpha = 0.34$ for all cases to get the effect of the capital share on welfare.
2. γ is set for each case to maintain the same initial TR across all cases.
3. $n_{d0} = 0.1$ and $\lambda_D = 0$.
4. Initial $\xi = 0.6$ and new $\xi = 0.5$ (10 percentage points drop in the iceberg costs).
5. f_0 and f_E are set to match the initial 22.3 percent exporter rate and the initial total mass of producers.

Long-Run Growth Rates Simulation Results Table 1 shows the long-run growth rates following a 10 percentage point iceberg cost cut based on the 3 methods: numerical approximation, exact solution, and log-linearization. In terms of the change in consumption, the exact solution yields an increase of 1.052 percent while our numerical approximation method overstates this by less than 0.002 percent (1.054 percent). The log-linearization underpredicts the gain with an increase of 0.885 percent.

As we remove elements from the model (materials and capital) we find that our approximation is quite close to the exact solution. The results show that our numerical approximation performs quite well in getting the right growth rates of key variables in this variation. The log-linearization method using the system of equations for the long-run equilibrium does not perform as well in getting the growth rates right.

Transition Dynamics With further restrictions we can show that there is overshooting following a cut in iceberg costs using log-linearization. Here, we show that this overshooting behavior is substantial and the log-linearization catches this behavior reasonably well. We also show that our approximation method can trace this overshooting along the transition with even higher precision. The additional assumption we make are no non-tradable good sector ($\gamma = 1$), no material inputs ($\alpha_m = 0,$),and complete depreciation of capital ($\delta = 1$). We can obtain the transition dynamics without approximating the distribution, value functions, or the exporting threshold. Figure 1 depicts the results from our approximation methods with the simulation without approximation to evaluate the results and shows that our approximation is quite accurate and does a bit better than the log-linear approximation.

Table 2 shows that our approximation is quite accurate. The gap in welfare gains, measured as

$$\ln\left[\sum_{t=1}^{50}\beta^t\left(c_t^{Exact}/c_0^{Exact}\right)^{1-\sigma}/\sum_{t=1}^{50}\beta^t\left(c_t^{Approx}/c_0^{Approx}\right)^{1-\sigma}\right]$$

where c_0 is the initial steady state consumption, over the first 50 years is only -0.099 percent. Over the transition path the root mean square error of the period by period welfare gains, measured as

$$\sqrt{\frac{1}{50}\sum_{t=1}^{50}\left[\ln\left(c_t^{Approx}/c_0^{Approx}\right)-\ln\left(c_t^{Exact}/c_0^{Exact}\right)\right]^2},$$

is 0.001 percent.

Overall, our approximation method captures the dynamics of the economy quite well. While we cannot evaluate the magnitude of the approximation error in our full-blown model, the findings here suggest our approximation may be quite accurate.

6. Robustness

In this section we consider the robustness of our findings along three dimensions. First, we examine the longer run transitions into and out of exporting rather than the single year transitions we have emphasized. At these longer horizons the sunk cost model gets the persistence of non-exporters about right but generates exporters that are not persistent enough. That is, the likelihood of a current exporter exporting T≥2 years hence is too low in the model, while the likelihood of a current non-exporter not exporting T≥2 years hence is about right. We propose a simple extension of this model that allows for a lower cost of re-entering the export market for past exporters. This brings the micro evidence more in line with the data. Second, we discuss how to make the model consistent with the evidence in Ruhl and Willis (08) that the intensity with which exporters export increase in time since entry to the export market. Finally, we examine how our results change when underlying shocks have a Pareto distribution.

Overall, in bringing the model in-line with the observed high re-entry rates and growing export intensity, we find that our results about the size of trade costs and aggregate response to a cut in tariffs are qualitatively similar. Moreover, adding re-entry costs or gradual growth in export intensity point to the investment aspect of exporting being even more important than in our baseline calibration.

Where possible we consider evidence based on US plants; however, as we lack access to the underlying data, in places we also summarize data for a balanced panel of Chilean plants or discuss evidence from other papers. We then consider the aggregate implications of some of these gaps.

A. Multi-year Export Entry and Exit Rates

Our emphasis has been on capturing the high persistence of exporting based on the low annual exit rate from exporting. However, these annual transitions out of exporting may actually understate the persistence of exporting if former exporters can more easily re-enter the export market. The literature is mixed on the significance of re-entry. For the US this seems to be the case as Bernard and Jensen (2004) find that plants that last exported two years ago are more likely to export controlling for a host of other current plant characteristics. In contrast, in their study of Colombian manufacturers, Roberts and Tybout (1997) find a small, but insignificant increase in the exporting likelihood of plants that last exported two or three years ago. Das, Roberts, and Tybout (2007) do not explicitly discuss re-entry rates. To the extent that re-entry rates are high, this thus suggests that either the costs of re-entering are lower or the benefit of exporting for a past exporter is higher than for a virgin exporter.[11]

In the paper we focus on matching the annual transitions into and out of exporting of US manufacturing plants. We could just as easily look at longer term entry and exit rates. To some degree, Bernard and Jensen (2004) study these longer term entry rates in a balanced panel of U.S. manufacturing plants from the LRD. These are the largest manufacturing plants in the US and thus quite a different sample from the sample used in their earlier paper. While BJ find substantial churning at the annual frequency, they also find past exporters are apt to reenter. For instance, they find on average 12.5 percent of exporters stop exporting from one year to the next but that there is only a 21 percent likelihood of a current exporter not exporting 8 years later. Likewise they find that in the typical year roughly 13.9 percent of non-exporters start exporting while 8 years later 30 percent are exporters. Using the entry and exit rates from the data to predict export and non-export status 8 years hence, BJ calculate that after 8 years there are not enough non-exporters among initial exporters (21 percent vs. 42 percent) and not enough exporters among initial non-exporters (30 percent and 65 percent respectively.) Thus the one-year transitions appear to substantially understate the persistence of export status. Figure 2 depicts the T year exporter transitions calculated by Bernard and Jensen (04) and in our benchmark model.

In comparing the findings here with the model, a few notes of caution are necessary. First, these findings are for a period in which US export participation rose roughly 20 percent within the panel. Indeed the statistic should be interpreted not as an 8 year horizon but the probability of exporting in 1992 given export participation in 1984.

Figure 2 shows that the model misses somewhat on the persistence of exporting. One possible reason it misses is that the sample in the model is different from the sample in the data, which is geared towards larger plants. To examine this idea, we select a sample of plants by employment size. Doing so, we find the model can actually now capture the long-horizon stopper rates but misses out on the long-horizon starter rates. In particular, in our model for plants with 100+ (250+) employees we find a one year stopper rate of 1.3 percent (1.0 percent) and an 8 year exit rate of nearly 31 percent (24 percent). Thus it appears that the model is somewhat consistent with the longer run exporter transitions of 21 percent in the data. Likewise, for plants with 100+ (250+) employees we find a one

[11] In our Chilean panel, we find that past exporters are more likely to re-enter the export market than those plants that did not previously export (at least within the time frame studied). Indeed, running a simple probit regression we find that lagged export status is a significant predictor of current export status at horizons from 1 to 8 years. That is, take two plants of the same size that have not exported for the last k years. If one exported k+1 years ago then it is more likely to export in the current year. Similarly, if we aggregate plants over two years instead of one, we find that exit rates from exporting are only slightly higher (12 percent vs 13 percent). A purely random exporting model would predict a much larger increase in exit rates (but obviously not a doubling). So, on the costs side, we definitely see that the likelihood of re-entering seems to fall with the time since last exporting.

On the benefits side, we can examine whether the export intensity upon entry of re-entrants is higher given more recent export participation. Here we find that plants who stop exporting for one year that come back start out with an export-sales ratio that is 5 percentage points higher. Beyond one year out of the market, we do not see big differences in initial export intensity.

year starter rate of 33 percent (47 percent) and an 8 year starter rate of 62.7 percent (75.2 percent) compared to 30 percent in the data. The model thus appears somewhat consistent with the longer-term stopper rates of large plants and less consistent with the longer-term starter rates of non-exporters. Of course, the one-year transitions are now off.

There are two main possible sources of the high re-entry rates. First, exporting could be mismeasured for exporters. That is, some exporters may indicate they are not exporting even if they are exporting. This would perhaps explain why exporters that have only been out of the market one year come back into the market with a much higher export intensity than other new exporters. Second, re-entry rates are indeed quite high. One way to rationalize high re-entry rates is for the costs of re-entering the export market to be lower than starting to export. We next develop a model with these re-entry costs.

A Model with Re-entry Costs

To capture the high re-entry rates we introduce a cost of re-entering the export market, denoted $f_R \leq f_0$. For simplicity, we assume that once a plant exports it can always re-enter the market by incurring the cost f_R following a non-exporting spell. As before a plant will be able to continue exporting by paying $f_1 \leq f_R$. This implies that the entire history of exporting matters, not just the current export status. To accommodate re-entry we introduce a new state variable $\overline{m}_t = \max\{m_{t-i}\}_{i=0}^{\infty}$ into the individual state of the producer. This allows for a parsimonious expansion of the producer's state space. The value of a producer is then

$$V_{T,t}(z,v,m,\overline{m}) = \max\left\{V_{T,t}^{1}(z,v,m,\overline{m}), V_{T,t}^{0}(z,v,m,\overline{m})\right\},$$

where the value if decides not to export in $t+1$ is

$$\begin{aligned} V_{T,t}^{0}(z,v,m,\overline{m}) &= \max \Pi_{T,t}(z,v,m) \\ &+ n_s(z) Q_t \left(\frac{P_{t+1}}{P_t}\right) \int_{v'} \int_{z'} V_{T,t}\left(z',v',m',\overline{m}\right) \phi\left(z'|z\right) \phi_v\left(v'\right) dz' dv', \end{aligned} \quad (209)$$

and the value if it decides to export in period $t+1$ is

$$\begin{aligned} V_{T,t}^{1}(z,v,m,\overline{m}) &= \max \Pi_{T,t}(z,v,0) - W_t e^{v} f(m,\overline{m}) \\ &+ n_s(z) Q_t \left(\frac{P_{t+1}}{P_t}\right) \int_{v'} \int_{z'} V_{T,t+1}\left(z',v',1,1\right) \phi\left(z'|z\right) \phi_v\left(v'\right) dz' dv', \end{aligned} \quad (210)$$

where

$$f(m,\bar{m}) = \begin{cases} f_1 & m = 1, \\ f_R & (m,\bar{m}) = (0,1), \\ f_0 & (m,\bar{m}) = (0,0). \end{cases}$$

Clearly, as before we have entry and exit thresholds $(z_1(v,m,\overline{m}), z_0(v,m,\overline{m}))$ that depend on the fixed cost shock along with the current and past export status.

To evaluate this model, we set $f_R = f_1$ and then chose f_0 and f_1 to yield the same one-year transition and export participation rates as before. We set f_R so that past and current exporters face the same export decision rather than to hit a particular target on re-entry rates since we do not have a good measure of these re-entry rates. We set σ_v to minimize the gap between the model export participation rate distribution and the data. The parameters are reported in Table 3 and the fit of the model is summarized in Table 4.

Figure 2 plots the transition rates of exporters and non-exporters in the ergodic distribution of this new variation along with our original benchmark and the data. At longer horizons, the model with a re-entry decision can capture the high persistence of exporting without missing too badly on the persistence of not exporting. Indeed, in the new model gets 72 percent of initial exporters also export 8 years hence compared to 79 percent in the data and about 36 percent in our benchmark formulation. Among initial non-exporters, the re-entry model predicts 79 percent will remain non-exporters 8 years hence compared to 71 percent in the data and 73 percent in the benchmark sunk-cost model.

In addition to altering the longer-term transitions in and out of exporting, introducing a relatively low cost of re-entry affect both our estimates of the magnitude of trade costs and the aggregate consequences of a trade liberalization (Tables 5 and 6). In terms of trade costs, we now find that the up-front costs of starting to export are even more important, accounting for about 41 percent of export profits each period rather than 25 percent in our original benchmark. The increased importance of startup costs arises since now when plants enter for the first time they are buying an option to export for f_1 in all subsequent periods rather than just along a single export spell. Given the high survival rates of most plants this means that this investment will pay off in many more future states of the world, and so the upfront cost must be considerably higher. In terms of the aggregate response, we find that trade responds even more than before expanding by 108 percent vs. 92 percent in our benchmark. Moreover, now both the steady state and transition gains are larger than our benchmark model and the amount of overshooting (measured by the gap between welfare and steady state consumption) is slightly smaller.

B. Starter Export Dynamics

A number of papers (Ruhl and Willis, 2008 and Eaton et al., 2008a, 2008b) present evidence that among new exporters that continue exporting[12], export intensity (exports/total sales) rises with time in the market. They also show that exit rates seem to be quite high initially. Our model abstracts from all heterogeneity in the export intensity as this is determined entirely by the variable trade costs, which is the same for all plants in our model. We can thus introduce heterogeneity in export intensity by extending our model to allow for heterogeneity in variable trade costs. To capture the rise in export intensity, the variable trade cost must be negatively correlated with an exporter's time in the market. We next present a simple model that captures these growth dynamics and consider the aggregate consequences of these growth dynamics for our findings on the size of trade costs and aggregate consequences of tariffs. The findings are quite similar to a calibration that makes exporting a more durable activity by lowering the exit rate from exporting (as in the sunk cost calibration with more persistent exporting) since plants that have low trade costs will give up more when they stop exporting.

A Model with New Exporter Dynamics

To capture the observed growth in the export intensity, we allow iceberg trade costs to vary with the time since a plant started exporting.[13] In particular we assume the iceberg cost is ξ. We assume new exporters draw their iceberg cost from $\phi_0^{\xi}(\xi)$ and that there is an exogenous transition probability from ξ to ξ' of $\phi_1^{\xi}(\xi'|\xi)$. Thus, new exporters will start with a low export intensity and will grow on average if the mean trade cost of the initial distribution is greater than the mean trade cost from the unconditional distribution.[14] Obviously with this structure of trade costs, the exit threshold $z_1(v, m, \xi)$ will be increasing in ξ so that plants with high trade costs will require a better technology to stay in the export market all else equal. The problem of a producer is then

$$V_{T,t}(z, v, m, \xi) = \max\left\{V_{T,t}^1(z, v, m, \xi), V_{T,t}^0(z, v, m, \xi)\right\}$$

[12] Ruhl and Willis (2008) focus on two features about exporter dynamics based on a panel of Colombian manufacturers. First, starters have high exit rates. Second starters start out small, so that exports account for a relatively small share of total sales. They take about 5 years to mature to the average export intensity of exporters. They conclude that the costs of starting to export may be low (as are the returns to exporting).

Eaton, Eslava, Kugler and Tybout (2008) consider export dynamics of Colombian firms, not just manufacturing plants, based on transaction level Customs data. They find that each year most exporters are new but account for very little of the change in trade. Most of these new exporters exit soon after entering. The exporters that continue tend to grow by selling more to existing markets and expanding the number of markets they export to. In related work (A Search and Learning Model of Export Dynamics, Eaton, Eslava, Krizan, Kugler and Tybout, 2008), they attribute these gradual movements in exporting to search frictions in finding buyers and learning about the firm's export ability. This model introduces greater uncertainty to the export decision than in our formulation of the sunk cost model where uncertainty is driven by the shocks to productivity and fixed trade costs.

[13] In general, anything that makes sales grow in a manner that is related to the time since a plant started exporting and is not perfectly correlated with plant productivity will generate the patterns in the data.

[14] An alternative approach would be to allow iceberg trade cost to depend on the time in the market ξ_t. Such an approach would substantially increase the state space without necessarily making much of a difference in the plants problem since uncertainty over the growth path of exports is of second-order concern to plants. Moreover, with the stochastic trade cost, we can also capture the lowering of export-sales ratios on the path to exit observed in the data.

where the value if it decides not to export in period $t+1$ is

$$V_{T,t}^{0}\left(z,v,m,\xi\right) = \max \Pi_{T,t}\left(z,v,m,\xi\right) + n_s\left(z\right) Q_t \left(\frac{P_{t+1}}{P_t}\right) \int_{v'} \int_{z'} V_{T,t}\left(z',v',0,\xi\right) \phi\left(z'|z\right) \phi_v\left(v'\right) dz' dv' \tag{211}$$

and the value if it decides to export in period $t+1$ is

$$V_{T,t}^{1}\left(z,v,m,\xi\right) = \max \Pi_{T,t}\left(z,v,m,\xi\right) - W_t e^{v} f_m + n_s\left(z\right) Q_t \left(\frac{P_{t+1}}{P_t}\right) \int_{\xi'} \int_{v'} \int_{z'} V_{T,t+1}\left(z',v',1,\xi'\right) \phi\left(z'|z\right) \phi_v\left(v'\right) \phi_m^{\xi}\left(\xi'|\xi\right) dz' dv' d\xi', \tag{212}$$

Note in this model that a plant that stops exporting both loses access to the export market as well as its variable cost of exporting. This will make it less desirable for a plant to exit the export market since re-entry will involve not only foregoing exporting with the low continuation cost but regress in its shipping technology.

To operationalize this idea we assume that starters start with ξ_H for the iceberg cost. In the following period, the exporters with ξ_H remain with ξ_H with probability of ρ_H, and move to ξ_L with probability $1-\rho_H$. Once a producer reaches ξ_L, it remains at ξ_L, Prob$(\xi_L|\xi_L) = 1$. The shock to iceberg costs occurs after the decision on which markets to serve has been made.

This modification introduces 3 new parameters (ξ_L, ξ_H, ρ_H). In choosing ξ_H/ξ_L we target an initial export intensity of 1/2 of the unconditional average exporter. We then choose ρ_H so that the average export intensity of continuing exporters is equal to the unconditional in 5 years. Note these are slightly higher initial export intensities than in Ruhl and Willis (2008). We choose this higher level since in the data an adjustment must be made in the first year of exporting to account for the partial year of exporting. Figure 3 shows how the mean export intensity evolves in this model.

We also changed γ to match the initial trade share, $\gamma = 0.222$. For all other parameters except f_0, f_1, and f_E, we used the ones in the benchmark. Table 3 includes the parameters.

Allowing export intensity to grow changes both our estimates of the nature of trade costs and the aggregate response to a cut in tariffs. With regards to trade costs, we now find that the resources consumed paying fixed costs of exporting are slightly more than in our benchmark - about 60 percent of export profits compared to 53 percent previously. The split into upfront costs and continuation costs though is somewhat different (16 and 45 now vs. 25 and 28 percent). This split though is somewhat misleading as plants in this environment take longer to mature and so part of the costs incurred as plants mature should be viewed as investments in future export profits similar to how the upfront cost in our benchmark model are viewed.

In terms of the aggregate response to a cut in tariffs, compared to our benchmark, we find a slightly weaker steady state increase in consumption (0.80 percent vs. 0.84 percent) and a stronger overshooting along (1.20 percent vs. 1.03 percent) with a considerably stronger trade response (164 percent vs. 92 percent). The aggregate response is similar to a variation of the sunk-cost model with a very high persistence of exporting. This is intuitive since in both variations the investment in exporting will pay off with a greater lag and the opportunity cost of exiting is relatively large. In the sunk cost model, this is all rolled into the gap between the upfront cost and the continuation cost, while with the growing export intensity the exporter also loses access to the efficient shipping technology.

C. Pareto Distribution

Here we consider how the nature of plant heterogeneity affects our results. Specifically, we consider the case in which productivity has a Pareto distribution rather than Log-Normal. While the Pareto distribution is commonly employed in analytical work, it is clearly a poor approximation of the manufacturing plant-size distribution (see Rossi-Hansberg and Wright, 2007) as too much employment is concentrated among the largest plants because of the fat tails of the Pareto distribution. Nonetheless we again find the aggregate response to a cut in tariffs depends on the size of startup and continuation costs.

We make the following assumptions about the nature of heterogeneity. Entrants draw from a Pareto distribution with pdf of $\phi_E(a) = \eta a^{-\eta-1}(a_{\min} - \mu_E)^{\eta}$. With probability ρ, incumbents draw a new productivity from the a different Pareto distribution of $\phi(a) = \eta a^{-\eta-1} a_{\min}^{\eta}$ and with probability $1-\rho$ they keep their previous productivity. We set $\eta = 1.1$ to be close to the establishment size distribution. By having entrants and incumbents draw from different distributions we can capture the slow growth of newborn plants.

Most aggregate parameters are identical or chosen in the same way as in our original calibration. In terms of the parameters governing plant dynamics we set $\lambda = 0$ so that the survival probability is independent of productivity. We then choose μ_E and $a_{\min}$ to get the same size disadvantage of entrants and match the establishment size distribution. The last two columns of Table 4 report the parameters for variations with (Sunk-Pareto) and without sunk costs (Fixed-Pareto) while Table 5 reports some characteristics of starters and stoppers and measures of trade costs.

Figure 4 shows that Pareto shocks do not accurately capture the plant size distribution. Now, too much employment is concentrated in the largest plants (over 40 percent compared to 10 percent in the data). This is the same whether or not there is a sunk aspect to exporting. The overall fit of the calibration is much worse with the root mean square error now between 11.2 to 11.8 percent compared to 1.5 percent in our benchmark calibration.

Table 6 reports the consequences of an unanticipated 8 percentage point cut in tariffs with a Pareto distribution. Considering the steady state changes following a cut in tariffs, we again find that the trade response (88.9 percent vs. 53.2 percent) is stronger in the sunk-cost model while the consumption increase is smaller (0.63 percent vs. 0.88 percent). Again, including the transition we find that the sunk-cost model generates a larger welfare gain than the model with just a fixed cost (0.97 percent vs. 0.74 percent).

References

[1] Bernard, A.B. and J.B. Jensen (2004): "Why Some Firms Export" *Review of Economics and Statistics,* 86, 561-69.

[2] Eaton, Jonathan Marcela Eslava, Maurice Kugler, and James Tybout (2008a) "Export Growth in Colombia: Firm-Level Evidence," in *The Organization of Firms in a Global Economy*, edited by Elhanan Helpman, Dalia Marin, and Thierry Verdier. Harvard University Press.

[3] Jonathan Eaton (2008b), Marcela Eslava, C. J. Krizan, Maurice Kugler, and James Tybout "A Search and Learning Model of Export Dynamics." mimeo.

[4] Rossi-Hansberg, Esteban and Mark Wright (2007). "Establishment Size Dynamics in the Aggregate Economy," American Economic Review, 97(5) 1639-66.

[5] Ruhl, Kim and Jonathan Willis (2008). "New Exporter Dynamics," mimeo

[6] Tauchen, George, (1986). "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," Economics Letters 20, 177-181.

Table 1: Long-Run Growth Rates Following a 10 Percentage Point Iceberg Cost Cut (in percent)

	C	TR	n_X	N_T	N_N
material+capital+labor					
Approximation	1.054	33.67	34.81	-1.17	0.00
Exact	1.052	33.65	34.82	-1.17	0.00
Log-linear	0.885	32.79	34.14	-1.35	0.00
capital+labor					
Approximation	0.580	33.67	34.81	-1.17	0.00
Exact	0.579	33.65	34.82	-1.17	0.00
Log-linear	0.487	32.79	34.14	-1.35	0.00
labor only					
Approximation	0.383	33.67	34.81	-1.17	0.00
Exact	0.382	33.65	34.82	-1.17	0.00
Log-linear	0.321	32.79	34.14	-1.35	0.00

Table 2: Accuracy on Welfare Gains over First 50 Years Following a 10 Percentage Point Iceberg Cost Cut (in percent)

	Percentage Difference	Root-Mean-Squared-Error
Approximation	-0.0983	0.0013
Log-linear	0.6213	0.0341

Table 3: Parameter Values

	Common Parameters				
	β	σ	θ	δ	τ
	0.96	2	5.0	0.10	0.08
	Model Parameters				
	Sunk-Cost	Re-Entry with f_1	Growing export intensity	Sunk-Pareto	Fixed-Pareto
α	0.286	0.284	0.284	0.220	0.218
λ	7.351	7.351	7.351	0	0
n_{d0}	0.022	0.022	0.022	0.023	0.023
α_x	0.804	0.804	0.804	0.804	0.804
γ	0.210	0.213	0.222	0.182	0.189
ξ	0.451	0.451	(ξ_L, ξ_H, ρ_H) $(0.077, 0.758, 0.970)$	0.451	0.451
ρ	0.655	0.655	0.655	0.655	0.655
σ_ε	0.333	0.333	0.333	-	-
μ_E	-0.353	-0.353	-0.353	-0.305	-0.305
$a_{\min}$	-	-	-	0.403	0.403
η	-	-	-	1.100	1.100
f_E	1.652	1.652	1.671	2.828	2.861
σ_v	1.104	0.300	1.104	1.104	1.617
f_0	0.342	0.528	0.216	0.213	0.061
f_1	0.018	0.010	0.026	0.013	0.061

Table 4: Target Moments

	Target value	Sunk-Cost	Re-Entry with f_1	Growing export intensity	Sunk-Pareto	Fixed-Pareto
5-year exit rate	0.370	0.370	0.370	0.370	0.110	0.110
Startups' labor share	0.015	0.015	0.015	0.015	0.006	0.006
Shutdowns' labor share	0.023	0.023	0.023	0.023	0.023	0.023
Stopper rate	0.170	0.170	0.170	0.170	0.17	0.742
Exporter ratio	0.223	0.223	0.223	0.223	0.223	0.223
Trade Share	0.039	0.039	0.039	0.039	0.039	0.039
Root mean squared error (%)						
Overall Fit	0	1.553	1.864	3.872	11.836	11.229
Establishments	0	0.376	0.384	0.447	13.941	13.960
Employment share	0	0.762	0.764	0.900	12.036	12.071
Export participation	0	2.489	3.002	6.297	7.048	3.382

Note: Overall fit is defined as the root mean squared error of establishment and export participation bins.

Table 5: Additional Implications

	Sunk-Cost	Re-Entry with f_1	Growing export intensity	Sunk-Pareto	Fixed-Pareto
Exporter Premium	3.50	3.38	3.13	4.78	4.41
Starter (employment)					
Mean	171.29	167.24	158.10	216.29	74.41
Median	60.85	21.77	54.70	17.19	11.23
Stopper (employment)					
Mean	13.38	3.24	23.51	13.72	23.56
Median	4.96	2.60	8.06	8.35	9.10
Startup Cost (1992 $ mill.)					
Mean	0.745	1.202 (new: 3.480, re: 0.089)	0.467	0.352	0.095
Median	0.639	3.680 (new: 3.243, re: 0.087)	0.419	0.226	0.098
Continuation Cost (1992 $ mill.)					
Mean	0.203	0.094	0.326	0.115	0.164
Median	0.136	0.087	0.207	0.088	0.098
Costs (% of Gross Export Profits)	52.98	53.45	60.36	27.18	19.14
Startup	25.27	40.58	15.75	11.29	12.10
Continuation	27.71	12.88	44.61	15.89	7.03
Startup Cost (% of)					
Mean profits of starters	99.08	177.17 (new: 218.44, re: 38.96)	125.48	43.40	27.26
Median profits of starters	388.24	2944.13 (new: 2289.24, re: 382.68)	16.28	504.04	165.14

Table 6: Percent Changes in Steady State and Transition Changes from Eliminating 8% Tariff

	Sunk-Cost	Re-Entry with f_1	Growing export intensity	Sunk-Pareto	Fixed-Pareto
Consumption	0.84	0.95	0.80	0.63	0.88
Trade to GDP ratio	92.31	108.14	164.07	88.88	53.17
Capital stock	1.06	1.17	1.02	0.84	1.09
Production labor	-0.26	-0.16	-0.27	-0.33	-0.12
Non-tradable variety	-0.45	-0.35	-0.46	-0.52	-0.30
Domestic tradable variety	-2.19	-3.32	-7.06	-2.37	0.63
Total tradable variety	11.13	12.14	13.60	22.43	3.68
Starter ratio	63.84	100.98	121.57	125.89	15.52
Stopper ratio	-35.51	-27.04	-48.22	-49.35	-5.91
Exporter ratio	74.66	87.75	121.87	139.31	16.67
Output premium	-1.97	11.36	-0.26	-36.46	0.24
Productivity premium	-10.30	2.14	-11.30	-30.52	12.28
Static welfare gains	0.84	0.95	0.80	0.63	0.88
Transitional welfare gains	1.03	1.07	1.20	0.97	0.74

Note: Welfare gains are measured as the value of x that satisfies $\sum_{t=0}^{\infty} \beta^t U\left(C_{-1}\left(1+x\right)\right) = \sum_{t=0}^{\infty} \beta^t U\left(C_t\right)$, where C_{-1} is the initial steady state consumption.

Figure 1: Transition Dynamics

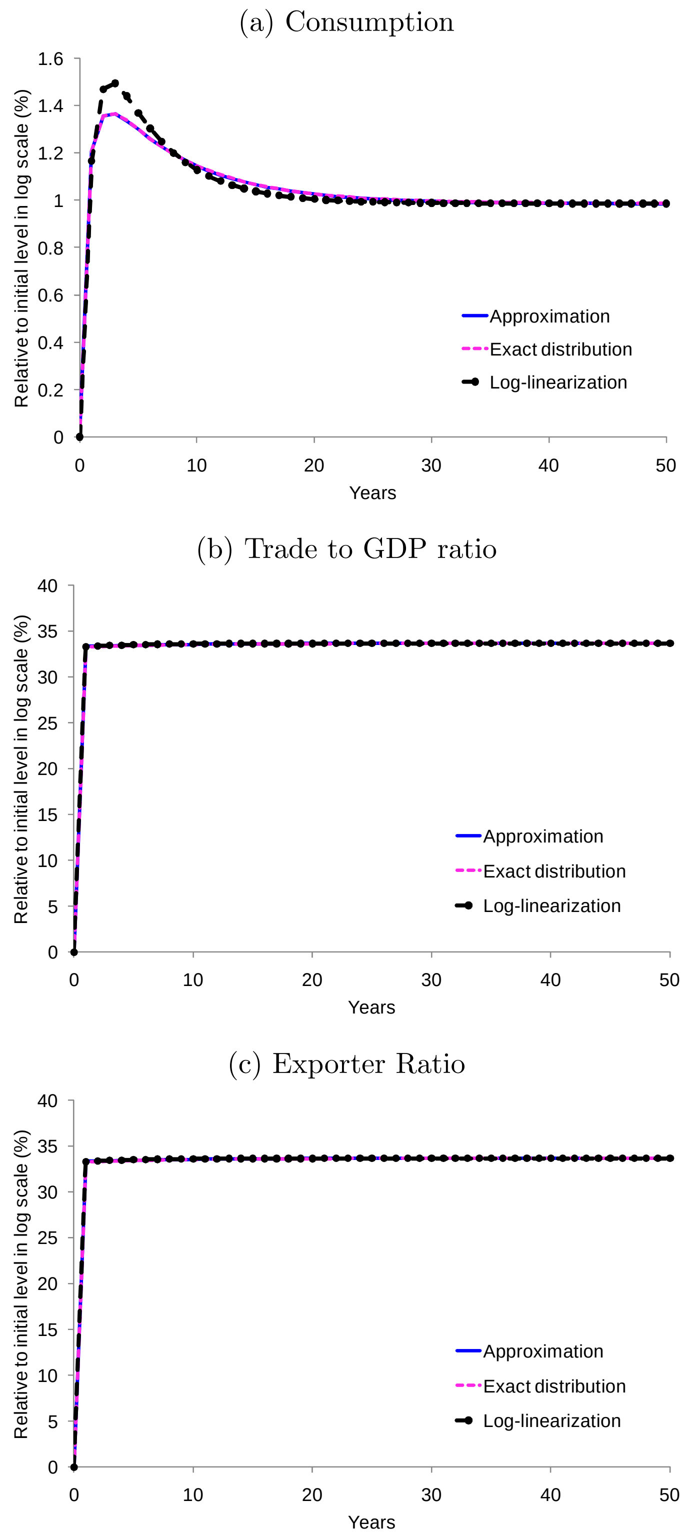
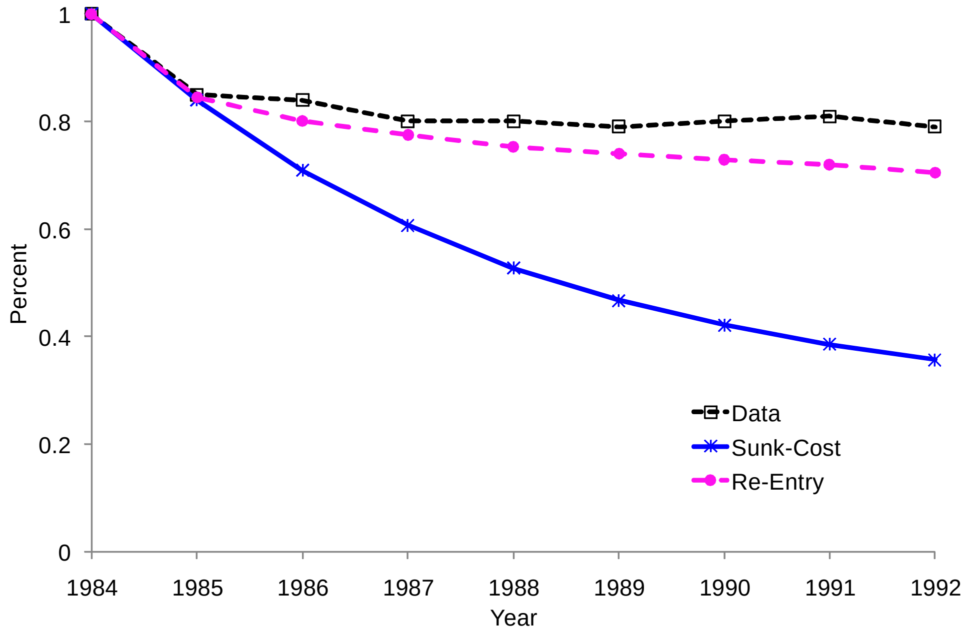


Figure 2: Exporter Transitions at Long-Horizons

(a) Share of Exporters among Initial Exporters



(a) Share of Non-Exporters among Initial Non-Exporters

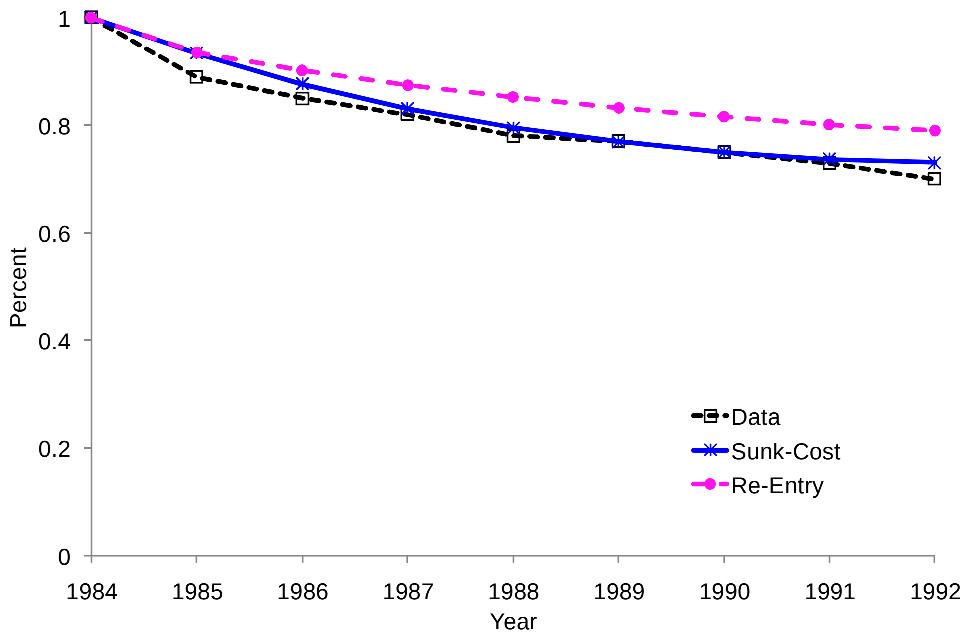


Figure 3: Growing Export Intensity Model

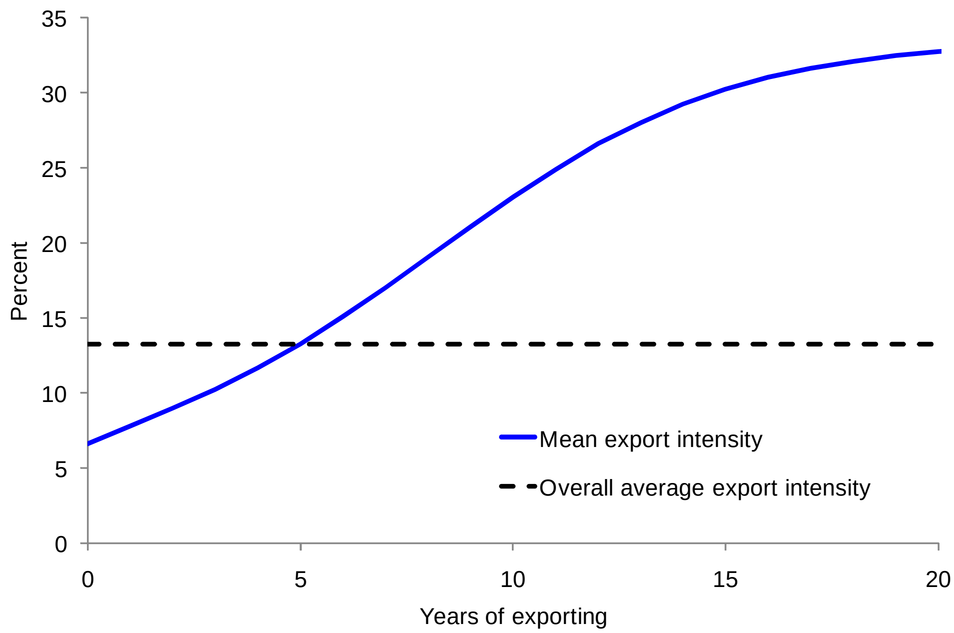
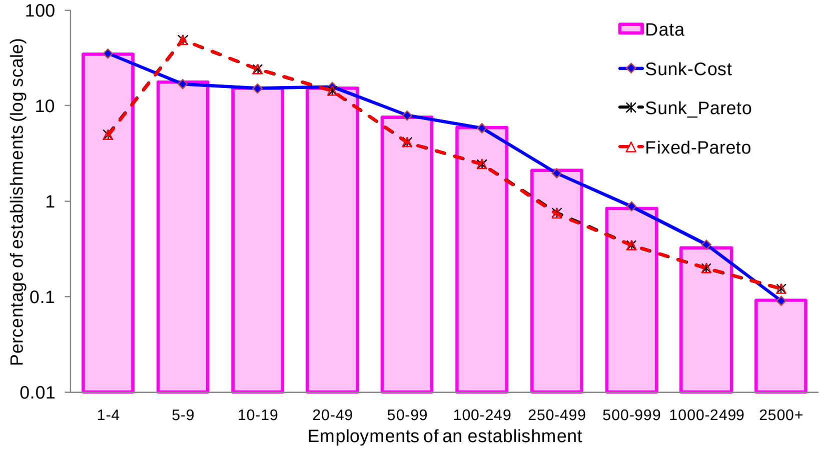
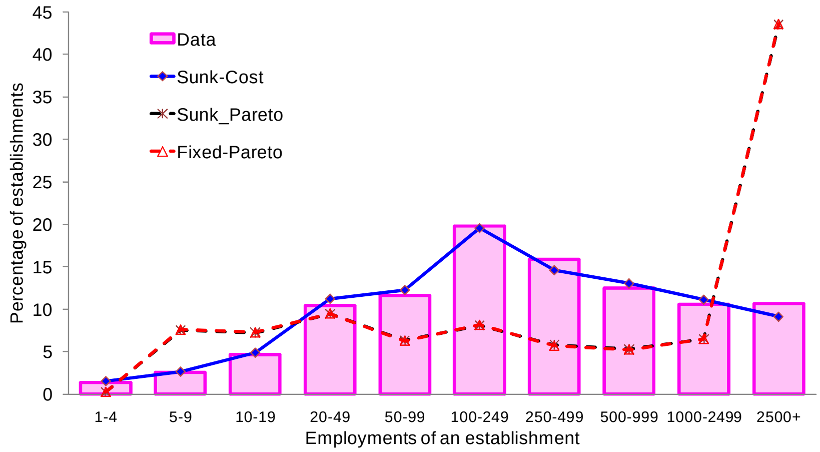


Figure 4: Employment Distributions

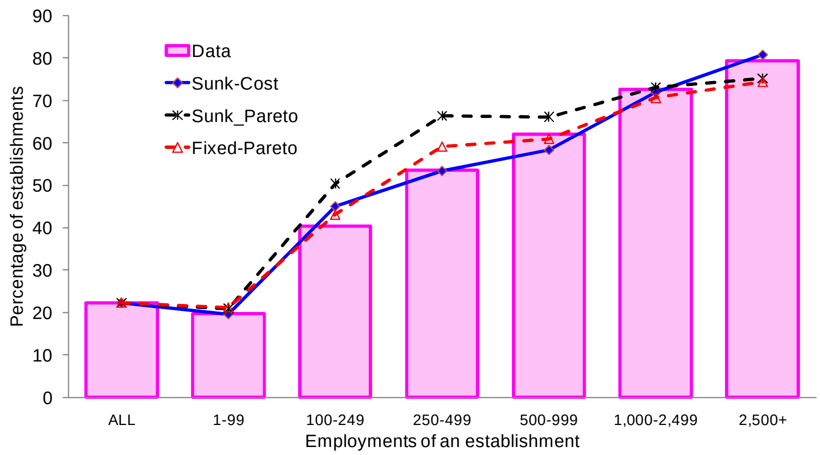
(a) Establishment Share



(b) Employment Share



(c) Export Participation



TECHNICAL APPENDIX II: KEY RESULTS WITH ANALYTICAL SOLUTIONS

(Not for Publication)

Establishment Heterogneity, Exporter Dynamics, and the Effects of Trade Liberalization*

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*The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

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These notes are divided in the following sections and subsections.

Contents

1. Analytical Solution from the Benchmark Model

In this section, we use the benchmark model, which is the most general model in our main text, to analytically derive some key results on long-run exporter, trade, and welfare growth. We show that the tariff distortion creates an inefficiency in allocation of resources. Eliminating this tariff distortion raises the capital to GDP ratio and lowers the consumption to GDP ratio. We also show that the thresholds for exporting are independent of production parameters, capital, and material input share parameters, once we calibrate the model to match the starter, stopper and exporter ratios. Thus, the exporter ratio is independent of the production parameters Lastly, we show that the growth rate of the trade share of GDP is independent of the capital share parameter, but the growth rate is increasing in the material input share parameter if the tariff rate is non-iceberg. However, if we cut the iceberg cost, the growth rate becomes independent of the material input share parameter.

To simplify notation, let $\tau^* = 1 + \tau$, $\xi^* = 1 + \xi$, and $a = e^{(\theta-1)z}$. We also shut down the fixed cost shock, $\sigma_v = 0$.

Let $S = \left(\overline{\Psi}_T + \tau^{*1-\theta}\xi^{*1-\theta}\overline{\Psi}_X\right) / \left(\overline{\Psi}_T + \tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X\right)$ measure the tariff distortion. This distortion measures the difference between the total market value of tradable goods and the total net revenue of tradable good producers. Note that $S > 1$ for any $\tau = \tau^* - 1 > 0$, and $S = 1$ for $\tau = \tau^* - 1 = 0$. This tariff distortion creates an inefficiency in the resource allocation of final goods.[1]

A. Tariff Distortion and Resource Allocations

Let's first examine how the tariff creates a distortion in resource allocation. The tariff rate works like a tax distortion in the world economy. From the capital stock in the steady state, we have the investment, and consumption to GDP ratios as

$$\frac{I}{D} = \delta\left(\frac{\theta-1}{\theta}\right)\left(\frac{\alpha}{R}\right)\left[(1-\gamma)+\gamma\left(\frac{1-\alpha_x}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right)\right], \quad (1)$$

$$\frac{C}{D} = 1-\delta\left(\frac{\theta-1}{\theta}\right)\left(\frac{\alpha}{R}\right)\left[(1-\gamma)+\gamma\left(\frac{1-\alpha_x}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right)\right]. \quad (2)$$

Since $S > 1$ for all $\tau > 0$ and $S = 1$ for $\tau = 0$, the elimination of tariffs raises the investment to GDP ratio, I/D, but lowers the consumption to GDP ratio, C/D.

Thus, when there is a cut in the tariff rate, the growth rate of consumption (investment or capital) is always lower (greater) than that of GDP. As tradable good producers make their investment based on their revenue not the market value of their products, a positive tariff rate creates under investment than the optimal level in the economy. The elimination of the distortionary tariff rate make the producers choose the optimal level of investment and capital. Thus, the investment to GDP ratio rises but consumption to GDP ratio falls following the elimination of the tariff rate.

[1] When there is only an iceberg cost, $S = 1$ for all tariff rates.

B. Independence of exporting thresholds

Let the normalized value of producers be

$$V_T^*(a,m) = V_T(a,m)\left(\frac{\theta}{\gamma}\right)\frac{N_T}{D}\left[S - \alpha_x\left(\frac{\theta-1}{\theta}\right)\right]. \tag{3}$$

Then, we can rewrite the value functions of a producer with (a,m) as

$$\begin{aligned} V_T^*(a,m) &= \left(\frac{1+m\tau^{*-\theta}\xi^{*1-\theta}}{\overline{\Psi}_T+\tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X}\right)a + \max\left\{\beta n_s(a)\int_{a'} V_T^*(a',0)\,\phi(a'|a)\,da',\ ,\right. \\ &\left.\beta n_s(a)\int_{a'} V_T^*(a',1)\,\phi(a'|a)\,da' - \beta n_s(a)\int_{a'}\left[V_T^*(a',1)-V_T^*(a',0)\right]\phi(a'|a_m)\,da'\right\}. \end{aligned} \tag{4}$$

The exporting thresholds solve

$$\left(\frac{\theta}{\gamma}\right)\frac{WN_T}{D}\left[S-\alpha_x\left(\frac{\theta-1}{\theta}\right)\right]f_m = \beta n_s(a_m)\int_{a'}\left[V_T^*(a',1)-V_T^*(a',0)\right]\phi(a'|a_m)\,da', \tag{5}$$

and the entry condition is

$$\left(\frac{\theta}{\gamma}\right)\frac{WN_T}{D}\left[S-\alpha_x\left(\frac{\theta-1}{\theta}\right)\right]f_E = \beta\int_{a'} V_T^*(a',0)\,\phi_E(a')\,da'. \tag{6}$$

Combining the equations for the export thresholds and the entry condition yields

$$\frac{f_m}{f_E} = \frac{n_s(a_m)\int_{a'}\left[V_T^*(a',1)-V_T^*(a',0)\right]\phi(a'|a_m)\,da'}{\int_{a'} V_T^*(a',0)\,\phi_E(a')\,da'}. \tag{7}$$

We set f_0/f_E and f_1/f_E to match the initial exporter ratio and stopper rate using (4) and (7). Since (4) and (7) are independent of production parameters, α and α_x, the calibrated values of f_0/f_E and f_1/f_E should be independent of the production parameters. The equations (4) and (7) also implies that changes in the export thresholds, a_0 and a_1, following a tariff cut should be independent of the production parameters. Thus, the growth rates of exporter, stopper and starter rates should be independent of the production parameters.

C. Independence of Trade Growth

The steady state trade share of GDP can be written as

$$TR = \left(\frac{\tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X}{\overline{\Psi}_T + \tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X} \right) \left(\frac{\gamma}{S - \alpha_x \left(\frac{\theta-1}{\theta}\right)} \right). \tag{8}$$

Thus, the growth rate of TR equals the growth rate of $\frac{\tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X}{\overline{\Psi}_T+\tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X}$ less the growth rate of $S - \alpha_x \left(\frac{\theta-1}{\theta}\right)$. From the results on the the growth rates of a_0 and a_1, we can find that the overall export intensity $\left(\frac{\tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X}{\overline{\Psi}_T+\tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X}\right)$ and the tariff distortion S are independent of the production parameters. Thus, the growth rate of TR is independent of the capital share parameter in production, α, but is increasing (in absolute value) in the material input share, α_x, as the growth rate of $S - \alpha_x \left(\frac{\theta-1}{\theta}\right)$ is increasing in α_x.

Note that when there are no tariff distortions in the economy, $S = 1$ for all tariff rates. In that case, the trade growth from a cut in iceberg costs is completely independent of the production parameters, α and α_x.

2. Fixed Cost Model with a Constant Exit Rate

It is not possible to analytically derive the welfare gains in the benchmark model without some further restrictions. Here, we work with the *Fixed Cost* model to isolate the key roles of the tariff distortion, positive discount rate, and disadvantage of entrants on the welfare gains.

The additional assumptions imposed in the model for analytical solutions are:

1. Producers can export their goods right away after paying fixed costs, $f_0 = f_1 = f_X$;
2. Constant shut down probability, $n_s(a) = n_s$; and
3. No entrant disadvantage (later we will discuss a case with the entrants' disadvantage). Specifically, we assume that the productivity distribution of entrants is the same as that of incumbents.

A. Parameterization for Key Moments

We exogenously set β, n_s, and τ^*. Given τ^*, we set ξ^* to match the initial export intensity $\tau^{*-\theta}\xi^{*1-\theta}/\left(1+\tau^{*-\theta}\xi^{*1-\theta}\right)$ of an exporter as in the data, Then, we set f_E/f_X to match the initial exporter ratio, n_X, equivalently a_X and $\overline{\Psi}_X$, as in the data. This calibration automatically pins down the initial total domestic revenue to total export revenue of tradable good producers, $g = \overline{\Psi}_T/\left(\tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X\right)$. It also automatically pins down the initial tariff distortion measure, S. Given a production function, we set γ to match the initial trade share, TR.

B. Trade Growth

Now, we are ready to analyze the effect of a cut in tariff rate on trade growth. From now on, we abuse notations a little: the variable without ˆ denotes the value of the variable in the initial steady state. With the log-linearization of the system of equations that governs the steady state of the economy around the initial steady state, we have

$$\widehat{a}_X = \left(\frac{\theta g}{g+1}\right)\widehat{\tau}^*, \tag{9}$$

where $\widehat{x} = dx/x$. As we saw in the general model solution, the growth rate of exporting threshold is independent of the discount factor and the production parameter. Thus, the exporter ratio is also independent of those parameters,

$$\begin{aligned} \widehat{n}_X &= -\frac{a_X \phi_X}{n_X^0}\widehat{a}_X \\ &= -\frac{a_X \phi_X}{n_X}\left(\frac{\theta g}{g+1}\right)\widehat{\tau}^*, \end{aligned} \tag{10}$$

where $\phi_X = \phi_a(a_X)$. From the definition of S, we have

$$\widehat{S} = \frac{\tau(g+1)^2 - \theta g(\tau^* - 1)\left[1 + g\left(1 + \frac{a_X^2 \phi_X}{\overline{\Psi}_X}\right)\right]}{(g+1)^2 (g+\tau)}\widehat{\tau}^*. \tag{11}$$

Thus, the tariff distortion is independent of production parameters and the time discount factor once we match the key moments.

Clearly, $\widehat{S}/\widehat{\tau}^*$ is not always positive. However, we can find that $\widehat{S}/\widehat{\tau}^*$ at $\tau^* = 1$ is always positive,

$$\left.\frac{\widehat{S}}{\widehat{\tau}^*}\right|_{\tau^*=1} = \frac{1}{g+1} > 0. \tag{12}$$

Furthermore, $\widehat{S}/\widehat{\tau}^*$ is monotonically decreasing in τ^*,

$$\frac{\partial\left(\widehat{S}/\widehat{\tau}^*\right)}{\partial \tau^*} = -\frac{g\left[(\theta-1)(g+1) + \theta g\left(1 + \frac{a_X^2 \phi_X}{\overline{\Psi}_X}\right)\right]}{(g+1)(g+\tau)^2} < 0. \tag{13}$$

So, for $1 \leq \tau^* \leq \overline{\tau}$, S is increasing τ^*, where $\overline{\tau} = \theta g\left[1 + g\left(1 + \frac{a_X^2 \phi_X}{\overline{\Psi}_X}\right)\right] / \left\{\left[1 + g\left(1 + \frac{a_X^2 \phi_X}{\overline{\Psi}_X}\right)\right] - (g+1)^2\right\}$. From now on, we will consider only $1 \leq \tau^* \leq \overline{\tau}$, where S is increasing in τ^*.

From trade to GDP ratio equation, we have

$$\widehat{TR} = -\left(\frac{g}{g+1}\right)\left(\theta\widehat{\tau}^{*} - \widehat{\overline{\Psi}}_X\right) - \frac{S}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\widehat{S} \tag{14}$$
$$= -\left(\frac{\theta g}{g+1}\right)\left[1+\left(\frac{g}{g+1}\right)\left(\frac{a_X^2\phi_X}{\overline{\Psi}_X}\right)\right]\widehat{\tau}^{*} - \frac{S}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\widehat{S}.$$

As we saw in the general case, the trade share growth rate is clearly independent of β and α. However, the trade growth is increasing in α_x. We can also find that the tariff distortion further raises the trade share growth.

C. Wage Rate to GDP Ratio

From the budget constraint, we have

$$\frac{WL}{D} = \frac{\gamma}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\left\{1-\left(\frac{\theta-1}{\theta}\right)\left[\alpha_x+\alpha\left(1-\alpha_x\right)\right]-\frac{1-\beta}{\theta\left(1-\beta n_s\right)}\right. \tag{15}$$
$$\left.+\frac{1-\beta}{\theta\left(1-\beta n_s\right)}\left[\frac{\tau^{*-\theta}\xi^{*1-\theta}n_X a_X}{\overline{\Psi}_T+\tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X}\right]\right\}+\left(1-\gamma\right)\left[1-\alpha\left(\frac{\theta-1}{\theta}\right)-\frac{1-\beta}{\theta\left(1-\beta n_s\right)}\right].$$

The first and second terms are the wage bill in the tradeable and non-tradable good sectors, respectively. Thus, both terms are always positive. In growth rates, we have

$$\left(\frac{WL}{D}\right)\left(\widehat{W}-\widehat{D}\right) = -TR\left(g+1\right)\left\{1-\left(\frac{\theta-1}{\theta}\right)\left[\alpha_x+\alpha\left(1-\alpha_x\right)\right]\right. \tag{16}$$
$$\left.-\frac{1-\beta}{\theta\left(1-\beta n_s\right)}\left[1-\frac{a_X n_X}{\left(g+1\right)\overline{\Psi}_X}\right]\right\}\left(\frac{S}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right)\widehat{S}$$
$$-\frac{TR\left(1-\beta\right)}{\theta\left(1-\beta n_s\right)}\left(\frac{\theta g}{g+1}\right)\left(\frac{a_X^2\phi_X}{\overline{\Psi}_X}\right)\left[1-\frac{a_X n_X}{\left(g+1\right)\overline{\Psi}_X}\right]\widehat{\tau}^{*}$$

Since $a_X n_X/\overline{\Psi}_X < 1$ and $g \geq 1$, the wage to GDP ratio is always increasing with a tariff cut for $1 \leq \tau^* \leq \overline{\tau}$. It is not clear whether the discount rate β raises or lowers the growth rate of wage to GDP ratio. Clearly $\left(1-\beta\right)/\left(1-\beta n_s\right)$ is decreasing in β with zero when $\beta = 1$. Thus, the second term (in absolute value) is always decreasing in β, that is, the lower the discount factor, the greater the growth rate of wage to GDP ratio following a tariff cut. The first term also implies that a tariff reduction raises the wage to GDP ratio through a fall in the tariff distortion, S. However, the first term (in absolute value) is increasing in β, that is, the response of wage to GDP ratio following a tariff cut is dampened with a lower β. To assess the overall effect of β on the growth rate, let's evaluate the growth rate at

$\tau^* = 1$. With $\tau^* = 1$, we have $\widehat{S} = (g+1)^{-1}\widehat{\tau}^*$ and $S = 1$. Thus we have

$$\left(\frac{WL}{D}\right)\left(\widehat{W} - \widehat{D}\right) = -TR\left[1 - \left(\frac{\theta - 1}{\theta}\right)\alpha\right]\widehat{\tau}^* \qquad (17)$$
$$+\frac{TR(1-\beta)}{\theta(1-\beta n_s)}\left[1 - \frac{a_X n_X}{(g+1)\overline{\Psi}_X}\right]\left[\frac{1}{1-\alpha_x\left(\frac{\theta-1}{\theta}\right)} - \left(\frac{\theta g}{g+1}\right)\left(\frac{a_X^2\phi_X}{\overline{\Psi}_X}\right)\right]\widehat{\tau}^*.$$

The first term in the last bracket is the gross revenue relative to final good sales revenue of tradable good producers. The second term is the elasticity of export sales to total sales ratio, $\tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X/\left(\overline{\Psi}_T + \tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X\right)$, with respect to the marginal exporting threshold, α_X, multiplied by the elasticity of substitution. Note that if $\theta \geq 2$, we have $\theta g/(g+1) > 1$ as $g \geq 1$. The term $a_X^2\phi_X/\overline{\Psi}_X$ is the elasticity of $\overline{\Psi}_X$ with respect to a_X. If the elasticity is not too small in relative concept to $\theta g/(g+1)$, the last bracket is negative. In that case, the lower the discount factor, the greater the effect of a tariff cut on the wage to GDP ratio.

D. Mass of Producers

The entry condition into the non-tradable good sector implies

$$\widehat{N}_N = \widehat{D} - \widehat{W}. \qquad (18)$$

Thus, the mass of non-tradable good producers always falls following a tariff cut. The discount rate affects the growth rate indirectly through $\left(\widehat{D} - \widehat{W}\right)$. A lower discount rate tends to lower the mass further.

From the exporting decision and the entry condition into the tradable good sector, we have

$$\widehat{N}_T = \frac{\theta g}{(g+1)^2}\frac{a_X^2\phi_X}{\overline{\Psi}_X}\widehat{\tau}^* + \left(\widehat{D} - \widehat{W}\right) - \left(\frac{S}{S - \alpha_x\left(\frac{\theta-1}{\theta}\right)}\right)\widehat{S}. \qquad (19)$$

The first two terms always lower the mass of tradable good producers following a tariff cut. But, a reduction of the tariff distortion raises the incentive to create a tradable good producers. Again, if we evaluate it at $\tau^* = 1$, we have

$$\widehat{N}_T = \left(\widehat{D} - \widehat{W}\right) + \frac{1}{g+1}\left[\frac{\theta g}{g+1}\frac{a_X^2\phi_X}{\overline{\Psi}_X} - \frac{1}{1-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right]\widehat{\tau}^*. \qquad (20)$$

Thus, if the elasticity of export sales to total sales ratio, $\tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X/\left(\overline{\Psi}_T + \tau^{*-\theta}\xi^{*1-\theta}\overline{\Psi}_X\right)$, with respect to the marginal exporting threshold, α_X, is not too small, the mass of tradable good producers falls more than the mass of the non-tradable good producers does. Similar to the mass of non-tradable good producers, the discount factor tends to magnify this effect through $\left(\widehat{D} - \widehat{W}\right)$.

E. Consumption to GDP Ratio

From the consumption to GDP ratio, we have

$$\begin{aligned} \widehat{C} - \widehat{D} &= \left(\frac{D}{C}\right)\delta\left(1-\alpha_x\right)\left(\frac{\alpha}{R}\right)\left(\frac{\theta-1}{\theta}\right)\frac{\gamma S}{\left[S-\alpha_x\left(\frac{\theta-1}{\theta}\right)\right]^2}\widehat{S} \\ &= \left(\frac{D}{C}\right)\delta\left(\frac{\theta-1}{\theta}\right)\left(\frac{\alpha}{R}\right)TR\left(g+1\right)\left[\frac{S\left(1-\alpha_x\right)}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right]\widehat{S}. \end{aligned} \tag{21}$$

Thus, following a tariff cut, the consumption share of GDP always falls due to a reduction in the tariff distortion.

F. Wage Rate

From the export decision, we have

$$\widehat{N}_T + \left[\widehat{\overline{\Psi}_T + \tau^{*1-\theta}\xi^{*1-\theta}\overline{\Psi}_X}\right] = -\frac{\theta}{g+1}\widehat{\tau}^* + \left(\widehat{D} - \widehat{W}\right) - \left(\frac{\alpha_x\left(\frac{\theta-1}{\theta}\right)}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right)\widehat{S}, \tag{22}$$

where $\widehat{[\cdot]}$ denotes the growth rate of $[\cdot]$. Thus, from the equation for the wage rate determination, we have

$$\begin{aligned} \left(\theta-1\right)\left(1-\alpha\right)\widehat{W} &= \left(1-\gamma\right)\widehat{N}_N + \frac{\gamma}{1-\alpha_x}\left\{\widehat{N}_T + \left[\widehat{\overline{\Psi}_T + \tau^{*1-\theta}\xi^{*1-\theta}\overline{\Psi}_X}\right]\right\} \\ &= \left(1-\gamma+\frac{\gamma}{1-\alpha_x}\right)\left(\widehat{D}-\widehat{W}\right) - \left(\frac{\gamma}{1-\alpha_x}\right)\left(\frac{\theta}{g+1}\right)\widehat{\tau}^* \\ &\quad -\left(\frac{\gamma}{1-\alpha_x}\right)\left(\frac{\alpha_x\left(\frac{\theta-1}{\theta}\right)}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right)\widehat{S} \\ &= \left(1-\gamma+\frac{\gamma}{1-\alpha_x}\right)\left(\widehat{D}-\widehat{W}\right) - TR\left(g+1\right)\left(\frac{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}{1-\alpha_x}\right)\left(\frac{\theta}{g+1}\right)\widehat{\tau}^* \\ &\quad -TR\left(g+1\right)\left[\frac{\alpha_x\left(\frac{\theta-1}{\theta}\right)}{1-\alpha_x}\right]\widehat{S}. \end{aligned} \tag{23}$$

The wage rate rises directly through a fall in the tariff rate, and rises indirectly through a fall in the tariff distortion. However, the growth rate is dampened due to a rise in the wage to GDP ratio. The discount rate affects the growth rate indirectly through the wage to GDP ratio growth rate. A lower discount rate tends to lower the growth rate of the wage rate through a further reduction in the growth rate of D/W.

G. Welfare Gains

The consumption growth rate can be found from

$$\begin{aligned}
\widehat{C} &= \left(\widehat{C}-\widehat{D}\right)+\left(\widehat{D}-\widehat{W}\right)+\widehat{W} \qquad (24)\\
&= \left(\frac{D}{C}\right)\delta\left(\frac{\alpha}{R}\right)\left(\frac{\theta-1}{\theta}\right)TR\left(g+1\right)\left[\frac{S\left(1-\alpha_x\right)}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right]\widehat{S}.\\
&+\left(\frac{D}{WL}\right)\left\{1-\left(\frac{\theta-1}{\theta}\right)\left[\alpha_x+\alpha\left(1-\alpha_x\right)\right]-\frac{1-\beta}{\theta\left(1-\beta n_s\right)}\right.\\
&\left.+\frac{(1-\beta)}{\theta\left(1-\beta n_s\right)\left(g+1\right)}\frac{n_X a_X}{\overline{\Psi}_X}\right\}TR\left(g+1\right)\left(\frac{S}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right)\widehat{S}\\
&+\left(\frac{D}{WL}\right)\frac{TR\left(1-\beta\right)}{\theta\left(1-\beta n_s\right)}\left(\frac{\theta g}{g+1}\right)\left(\frac{a_X^2\phi_X}{\overline{\Psi}_X}\right)\left[1-\frac{a_X n_X}{\left(g+1\right)\overline{\Psi}_X}\right]\widehat{\tau}^*\\
&+\frac{1}{\left(\theta-1\right)\left(1-\alpha\right)}\left(1-\gamma+\frac{\gamma}{1-\alpha_x}\right)\left(\widehat{D}-\widehat{W}\right)-\frac{1}{\left(\theta-1\right)\left(1-\alpha\right)}\left(\frac{\gamma}{1-\alpha_x}\right)\left(\frac{\theta}{g+1}\right)\widehat{\tau}^*\\
&-\frac{1}{\left(\theta-1\right)\left(1-\alpha\right)}\left(\frac{\gamma}{1-\alpha_x}\right)\left(\frac{\alpha_x\left(\frac{\theta-1}{\theta}\right)}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right)\widehat{S}.
\end{aligned}$$

This equation is quite complicated to figure out exact effects on the consumption growth coming from various sources. From the very first equation, we can say that following a tariff cut, consumption rises with a rise in wage rate, however the growth rate is dampened due to a fall in C/D which is coming from the tariff distortion and reduced further due to a rise in wage to GDP ratio. The discount factor affects the consumption growth rate through the growth rate of D/W and W. A lower discount factor tends to lower the growth rate of consumption through a further fall in $\widehat{D}-\widehat{W}$ and a dampening effect on $\widehat{W}$.

This result is quite different from the case where there are no tariffs and we cut the iceberg cost and discount factor is one. In that case, the first and the second terms in the very first equation is zero, that is, the consumption growth rate is the same as the wage growth rate.

3. Roles of Capital and Material Inputs in Welfare Gains

In this section, we focus on the role of capital and material inputs in welfare gains. The main finding from a simplified model is that both the capital and material input share parameters raise the welfare gains from a cut in tariff rate.

It is almost impossible to figure out the roles of the capital and material inputs in welfare gains with a general model or a simplified model described above. To get a clearer idea about the roles, we simplify the model further. Let's consider case where there is only and iceberg cost. So, we are assuming $\tau^*=1$, and the tariff cut is coming from a cut in ξ^*. We further assume that a follows a Pareto distribution $\phi\left(a\right)=\eta a^{-1-\eta}$ and let $\beta\rightarrow 1$.

From the exporting and entry decisions, we have

$$\widehat{a}_X = \frac{g(\theta-1)}{g+1}\widehat{\xi}^*. \tag{25}$$

Thus, a cut in the iceberg tariff rate always raises the exporter ratio and the growth rate is independent of production parameters. We can also find that

$$\widehat{C} = \widehat{D} = \widehat{K} = \widehat{W}, \tag{26}$$

as there are no discount factor effects or the tariff distortion.

Since the mass of non-tradable good producers is proportional to D/W, the mass of non-tradable good producers do not change in the long-run.

$$\widehat{N}_N = 0. \tag{27}$$

Using the exporting decisions, we have

$$\widehat{N}_T = \frac{g(\theta-1)(\eta-1)}{(g+1)^2}\widehat{\xi}^*. \tag{28}$$

A cut in the iceberg tariff rate lowers the mass of producers in the tradable good sector. With these results together with the price normalization, we have

$$\widehat{C} = \widehat{W} = -\frac{\gamma}{(g+1)(1-\alpha)(1-\alpha_x)}\widehat{\xi}^*. \tag{29}$$

In the calibration we set γ to match the initial trade share $\gamma = TR^*(g+1)\left[1-\alpha_x\left(\frac{\theta-1}{\theta}\right)\right]$ yields,

$$\widehat{W} = -TR^*\left[\frac{1-\alpha_x\left(\frac{\theta-1}{\theta}\right)}{1-\alpha_x}\right]\frac{1}{(1-\alpha)}\widehat{\xi}^*. \tag{30}$$

Thus, the wage rate always rises with a fall in ξ^* and the growth rate is increasing in α and α_x.

The growth rate of TR is given as

$$\widehat{TR} = -\left(\frac{g}{g+1}\right)(\theta-1)(\eta g+1)\widehat{\xi}^*. \tag{31}$$

As shown in the general case, when the tariff rate is an iceberg, the growth rate of the trade share, TR, and the domestic sales to export sales ratio in the tradable good sector, g, is completely independent of the production parameters once we calibrate the model to match the initial g.

Since consumption is proportional to wage rate, we have

$$\widehat{C} = -TR\left[\frac{1-\alpha_x\left(\frac{\theta-1}{\theta}\right)}{1-\alpha_x}\right]\frac{1}{(1-\alpha)}\widehat{\xi}^{*}. \tag{32}$$

The consumption growth rate is increasing in capital and material input share parameters, α and α_x, but these parameters do not affect the trade growth once we match the initial TR and g.

4. Role of Entrants' Disadvantage

So far we assumed that the entrants do not have any disadvantage relative to incumbents. Here, we examine how the entrants' disadvantage affects the economy. Specifically we show that when $0 < \beta < 1$, the disadvantage works like a time discount factor in aggregate. Thus, the disadvantage of entrants alters the growth rate of consumption, wage rate, masses of producers in the tradable and non-tradable good sectors. However, this disadvantage effect vanishes when $\beta = 1$.

To make the model tractable, we add one more feature of the productivity process to *Fixed Cost* model. We assume that an entrant initially starts with a low productivity

$$a = a_l\varepsilon, \tag{33}$$

and it may make a permanent transition to

$$a = a_h\varepsilon, \tag{34}$$

where $a_h > a_l$ and ε follows an iid process. The transition probability is given as $\Pr(a_l|a_l) = \rho$, and $\Pr(a_l|a_h) = 0$. Let's further assume that producers with a_l do not export. For the probability of death, let's assume that the shut down probability depends on a. Producers with a_l and a_h face the survival probabilities of n_{sl} and n_{sh} $(> n_{sl})$. The masses of producers evolve as

$$N_{L,t} = N_{E,t-1} + n_{sl}\rho N_{L,t-1}, \tag{35}$$

$$N_{H,t} = n_{sl}(1-\rho)N_{L,t-1} + n_{sh}N_{H,t-1}, \tag{36}$$

where N_L, N_H, and N_E are the masses of producers with a_l, a_h and entrants, respectively. In the steady state, we have

$$N_L = \left(\frac{1}{1-n_L\rho}\right) N_E, \tag{37}$$

$$N_H = \frac{n_{sl}(1-\rho)}{(1-n_{sl}\rho)(1-n_{sh})} N_E, \tag{38}$$

$$N_T = \frac{n_{sl}(1-\rho)+(1-n_{sh})}{(1-n_{sl}\rho)(1-n_{sh})} N_E. \tag{39}$$

Let s_l and s_h be the shares of producers with a_l and a_h, respectively,

$$s_l = \frac{1-n_{sh}}{n_{sl}(1-\rho)+(1-n_{sh})}, \tag{40}$$

$$s_h = \frac{n_{sl}(1-\rho)}{n_{sl}(1-\rho)+(1-n_{sh})}. \tag{41}$$

With this productivity process, we have

$$\overline{\Psi}_T = [s_l a_l + s_h a_h] \int_\varepsilon \varepsilon \phi_\varepsilon(\varepsilon)\, d\varepsilon, \tag{42}$$

$$\overline{\Psi}_X = s_h a_h \int_{\varepsilon_0}^{\infty} \varepsilon \phi_\varepsilon(\varepsilon)\, d\varepsilon, \tag{43}$$

where $\phi_\varepsilon(\varepsilon)$ is the pdf of ε, and ε_0 is the exporting decision threshold where a producer with $\varepsilon = \varepsilon_0$ is indifferent from exporting and not exporting. Let

$$b = \int_\varepsilon \varepsilon \phi_\varepsilon(\varepsilon)\, d\varepsilon,$$

$$b_0 = \int_{\varepsilon_0}^{\infty} \varepsilon \phi_\varepsilon(\varepsilon)\, d\varepsilon.$$

The average values of producers in the non-tradable good sector for producers with a_l and a_h in the steady state are given as

$$V_{NL} = \left(\frac{1-\gamma}{\theta}\right) \frac{D}{N_N \overline{\Psi}_T} b a_l + \beta n_{sl} \left[\rho V_{NL} + (1-\rho) V_{NH}\right], \tag{44}$$

$$V_{NH} = \left(\frac{1-\gamma}{\theta}\right) \frac{D}{N_N \overline{\Psi}_T} b a_h + \beta n_{sh} V_{NH}. \tag{45}$$

Rearranging these equations, we have

$$V_{NL}=\frac{(1-n_{sh})+n_{sl}(1-\rho)}{(1-\beta n_{sh})(1-\beta\rho n_{sl})}\left[\frac{(1-\beta n_{sh})+\beta n_{sl}(1-\rho)\frac{a_h}{a_l}}{(1-n_{sh})+n_{sl}(1-\rho)\frac{a_h}{a_l}}\right]\left(\frac{1-\gamma}{\theta}\right)\frac{D}{N_N}. \tag{46}$$

The condition for the entry into the non-tradable good sector is

$$N_{NE}Wf_E=\frac{\beta(1-n_{sh})(1-\rho n_{sl})}{(1-\beta n_{sh})(1-\beta\rho n_{sl})}\left[\frac{(1-\beta n_{sh})+\beta n_{sl}(1-\rho)\frac{a_h}{a_l}}{(1-n_{sh})+n_{sl}(1-\rho)\frac{a_h}{a_l}}\right]\left(\frac{1-\gamma}{\theta}\right)D, \tag{47}$$

where $N_{NE}=N_N(1-n_{sh})(1-\rho n_{sl})/[(1-n_{sh})+n_{sl}(1-\rho)]$, the mass of entrants in the non-tradable good sector. Thus, the disadvantage of entrants works like a time discount factor, not necessary lowering the discount factor, in the aggregate economy for the steady state as long as $0<\beta<1$. When $\beta=1$, the disadvantage effect vanished in the non-tradable good sector.

The average values of producers in the tradable good sector for producers with a_l and a_h in the steady state are given as

$$\begin{aligned}V_{TL} &= \left[\frac{(1-n_{sh})+n_{sl}(1-\rho)}{(1-\beta n_{sh})(1-\beta\rho n_{sl})}\right]\left(\frac{\gamma}{\theta}\right)\left(\frac{SD}{N_T\left[S-\alpha_x\left(\frac{\theta-1}{\theta}\right)\right]}\right) \qquad (48)\\ &\left\{\frac{b\left[(1-\beta n_{sh})+\beta n_{sl}(1-\rho)\frac{a_h}{a_l}\right]+\beta n_{sl}(1-\rho)(1+\tau)^{-\theta}(1+\xi)^{1-\theta}b_0\frac{a_h}{a_l}}{b\left[(1-n_{sh})+n_{sl}(1-\rho)\frac{a_h}{a_l}\right]+n_{sl}(1-\rho)(1+\tau)^{1-\theta}(1+\xi)^{1-\theta}b_0\frac{a_h}{a_l}}\right\}\\ &-\left[\frac{\beta n_{sl}(1-\rho)}{(1-\beta n_{sh})(1-\beta\rho n_{sl})}\right]Wf_0(1-\Phi_{\varepsilon 0}),\\ V_{TH} &= \left[\frac{(1-n_{sh})+n_{sl}(1-\rho)}{1-\beta\rho n_{sl}}\right]\left(\frac{\gamma}{\theta}\right)\left(\frac{SD}{N_T\left[S-\alpha_x\left(\frac{\theta-1}{\theta}\right)\right]}\right) \qquad (49)\\ &\left\{\frac{\left[b+b_0(1+\tau)^{-\theta}(1+\xi)^{1-\theta}\right]\frac{a_h}{a_l}}{b\left[(1-n_{sh})+n_{sl}(1-\rho)\frac{a_h}{a_l}\right]+b_0(1+\tau)^{1-\theta}(1+\xi)^{1-\theta}n_{sl}(1-\rho)\frac{a_h}{a_l}}\right\}\\ &-\left(\frac{1}{1-\beta\rho n_{sl}}\right)Wf_0(1-\Phi_{\varepsilon 0}),\end{aligned}$$

where $1-\Phi_{\varepsilon 0}=\int_{\varepsilon_0}^{\infty}\phi_{\varepsilon}(\varepsilon)\,d\varepsilon$. The condition for the entry into the tradable good sector is given as

$$\begin{aligned} N_{TE}Wf_E &= \frac{\beta\left(1-n_{sh}\right)\left(1-\rho n_{sl}\right)}{\left(1-\beta n_{sh}\right)\left(1-\beta\rho n_{sl}\right)}\left(\frac{\gamma}{\theta}\right)\left(\frac{SD}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right) \\ &\left\{\frac{b\left[\left(1-\beta n_{sh}\right)+\beta n_{sl}\left(1-\rho\right)\frac{a_h}{a_l}\right]+\beta n_{sl}\left(1-\rho\right)\left(1+\tau\right)^{-\theta}\left(1+\xi\right)^{1-\theta}b_0\frac{a_h}{a_l}}{b\left[\left(1-n_{sh}\right)+n_{sl}\left(1-\rho\right)\frac{a_h}{a_l}\right]+n_{sl}\left(1-\rho\right)\left(1+\tau\right)^{1-\theta}\left(1+\xi\right)^{1-\theta}b_0\frac{a_h}{a_l}}\right\} \\ &-\left[\frac{\beta\left(1-n_{sh}\right)\left(1-\rho n_{sl}\right)}{\left(1-\beta n_{sh}\right)\left(1-\beta\rho n_{sl}\right)}\right]Wf_0N_T\left[\frac{\beta n_{sl}\left(1-\rho\right)}{\left(1-n_{sh}\right)+n_{sl}\left(1-\rho\right)}\right]\left(1-\Phi_{\varepsilon 0}\right). \end{aligned} \tag{50}$$

Again, the disadvantage of entrants works like a discount factor in the aggregate economy in the steady state.

For a special case where $\beta=1$, we have entry conditions as

$$N_{NE}Wf_E = \left(\frac{1-\gamma}{\theta}\right)D, \tag{51}$$

$$N_{TE}Wf_E+Wf_0N_T\left(1-\Phi_0\right) = \left(\frac{\gamma}{\theta}\right)\left(\frac{D}{S-\alpha_x\left(\frac{\theta-1}{\theta}\right)}\right). \tag{52}$$

Thus, when $\beta=1$, the aggregate economy in the steady state is independent of the entrants' disadvantage.

5. Transition Dynamics and Overshooting Behavior

In this section, we derive the transition dynamics of the economy in a simple model with a fixed cost of exporting. With this simple model, we can show analytically that the model generates an overshooting behavior following a tariff cut and the main driver of the overshooting is the transition of the mass of producers. For comparison's sake, we also work on the *No-Cost* model in which the fixed cost of exporting is zero and all producers are exporters. In that case, the economy makes a monotonic transition toward the new steady state without overshooting.

To derive the analytical solution, we use there is only an iceberg cost and assume further that capital completely depreciates, $\delta=1$, there are no material inputs in production, $\alpha_x=0$, and consider the case with $\beta\to 1$. We use the log utility function and a Pareto distribution with pdf of $\phi(a)=\eta a^{-1-\eta}$. The labor supply is normalized to 1, $L=1$.

The model has the following equations for equilibrium.

$$\begin{aligned}
K_{t-1} &= \left(\frac{\alpha}{R_t}\right)\left(\frac{\theta-1}{\theta}\right)D_t, & (53)\\
W_t f_x &= \frac{1}{\theta}\left(\frac{\theta MC_t}{\theta-1}\right)^{1-\theta}\xi^{1-\theta}a_{xt}D_t, & (54)\\
W_t f_E &= \left(\frac{C_t}{C_{t+1}}\right)V_{t+1}, & (55)\\
V_t &= \frac{1}{\theta}\left(\frac{\theta MC_t}{\theta-1}\right)^{1-\theta}D_t\left(\overline{\Psi}_T+\left(\frac{1}{\eta}\right)\xi^{1-\theta}\overline{\Psi}_{Xt+1}\right)+n_s\left(\frac{C_t}{C_{t+1}}\right)V_{t+1}, & (56)\\
1 &= \left(\frac{\theta MC_t}{\theta-1}\right)^{1-\theta}N_t\left(\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_{Xt}\right), & (57)\\
MC_t &= \left(\frac{R_t}{\alpha}\right)^{1-\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}, & (58)\\
1 &= \left(\frac{C_t}{C_{t+1}}\right)R_{t+1}, & (59)\\
D_t &= C_t+K_t, & (60)\\
D_t &= W_tL+R_tK_{t-1}+\frac{1}{\theta}\left(\frac{\theta MC_t}{\theta-1}\right)^{1-\theta}N_tD_t\left(\overline{\Psi}_T+\left(\frac{1}{\eta}\right)\xi^{1-\theta}\overline{\Psi}_{Xt+1}\right)-N_{Et}W_tf_E, & (61)\\
N_{t+1} &= n_sN_t+N_{Et}. & (62)
\end{aligned}$$

For the capital stock and consumption, we have

$$\begin{aligned}
1 &= \left(\frac{C_t}{C_{t+1}}\right)R_{t+1} \qquad (63)\\
&= \left(\frac{C_t}{C_{t+1}}\right)\left(\frac{\alpha}{K_t}\right)\left(\frac{\theta-1}{\theta}\right)(C_{t+1}+K_{t+1})\\
&= \alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{C_t}{K_t}\right)\left(1+\frac{K_{t+1}}{C_{t+1}}\right).
\end{aligned}$$

This gives

$$\frac{K_t}{C_t}=\alpha\left(\frac{\theta-1}{\theta}\right)+\alpha\left(\frac{\theta-1}{\theta}\right)\frac{K_{t+1}}{C_{t+1}}. \qquad (64)$$

Since $0<\alpha\left(\frac{\theta-1}{\theta}\right)<1$, we should have a constant K_t/C_t for stationarity. The ratio is given as

$$\frac{K_t}{C_t}=\frac{\alpha(\theta-1)}{\theta-\alpha(\theta-1)}. \qquad (65)$$

Since $D_t = C_t + K_t$,

$$C_t = \left[1 - \frac{\alpha(\theta-1)}{\theta}\right] D_t, \tag{66}$$

$$K_t = \frac{\alpha(\theta-1)}{\theta} D_t. \tag{67}$$

A. No-Cost Model

In *No-Cost* model, we have $f_x = 0$, and $a_{xt} = 1$ where all producers export. Thus, we have $\overline{\Psi}_{Xt} = \overline{\Psi}_T$, and $\frac{1}{\eta}\overline{\Psi}_{Xt}$ is replaced with $\overline{\Psi}_T$ as $f_x = 0$.

Steady State

In the steady state, we have $R = 1$, and

$$C = W. \tag{68}$$

Thus, consumption, capital, and output are always proportional to the wage rate.

From the entry decision

$$1 = \frac{1}{f_E\theta(1-n_s)} \left(\frac{\theta MC}{\theta-1}\right)^{1-\theta} \left(\frac{D}{C}\right) \left(\frac{C}{W}\right) \overline{\Psi}_T \left(1+\xi^{1-\theta}\right). \tag{69}$$

With the price normalization

$$N = \left[\frac{\theta}{\theta-\alpha(\theta-1)}\right] \frac{1}{f_E\theta(1-n_s)}. \tag{70}$$

Thus, the mass of producers is independent of the marginal trade cost.

Long-Run Growth Rates

From the budget constraint, we have

$$\widehat{D} = \widehat{C} = \widehat{K} = \widehat{W}, \tag{71}$$

so the long-run growth rates of GDP, consumption, capital, wage rate are the same.

From the price normalization, we have

$$\widehat{N} = 0, \tag{72}$$

the mass of producers do not change following a tariff cut.

The marginal cost in production gives

$$\widehat{MC} = (1-\alpha)\,\widehat{W}. \tag{73}$$

From the entry condition, we have

$$(1-\alpha)\,\widehat{W} = -\lambda_D \widehat{\xi}, \tag{74}$$

where $\lambda_D = 1/\left(1+\xi^{1-\theta}\right)$, domestic sales to GDP ratio. Thus, the long-run welfare gains are

$$\widehat{C} = -\left(\frac{1-\lambda_D}{1-\alpha}\right)\widehat{\xi}. \tag{75}$$

As we saw in the *Fixed Cost* model, the welfare gains are increasing in α once we calibrate the model to match the initial trade share of GDP, $TR = 1-\lambda_D$.

The growth rate of TR is given as

$$\widehat{TR} = -(\theta-1)\,\lambda_D \widehat{\xi}. \tag{76}$$

Thus, the trade growth is independent of α once we match the initial trade share of GDP.

Transition Dynamics

Now, let's log-linearize the system of equations around the new steady state. From the results for the capital-consumption ratio, we have

$$\widehat{D}_t = \widehat{K}_t = \widehat{C}_t. \tag{77}$$

From the capital share equation, we have

$$\begin{aligned} \widehat{R}_t &= \widehat{D}_t - \widehat{K}_{t-1} \\ &= \widehat{K}_t - \widehat{K}_{t-1}. \end{aligned} \tag{78}$$

From the value function and the entry condition, we have

$$W_t f_E = \left(\frac{C_t}{C_{t+1}}\right)\frac{1}{\theta}\frac{D_{t+1}}{N_{t+1}} + n_s\left(\frac{C_t}{C_{t+1}}\right)W_{t+1} f_E. \tag{79}$$

Dividing both sides with $C_t f_E$,

$$\left(\frac{W_t}{C_t}\right) = \frac{1}{\theta f_E}\left(\frac{D_{t+1}}{C_{t+1}}\right)\frac{1}{N_{t+1}} + n_s\left(\frac{W_{t+1}}{C_{t+1}}\right) \tag{80}$$

Log-linearization gives

$$\widehat{X}_t = (1-n_s)\,\widehat{N}_{t+1} + n_s\widehat{X}_{t+1}, \tag{81}$$

where $\widehat{X}_t = \widehat{K}_t - \widehat{W}_t$. From the budget constraint, we have

$$C_t = W_t + \frac{D_t}{\theta} - N_{Et}W_t f_E. \tag{82}$$

Dividing both sides with C_t, we have

$$1 = \left(\frac{W_t}{C_t}\right) + \frac{1}{\theta}\left(\frac{D_t}{C_t}\right) - N_{Et}\left(\frac{W_t}{C_t}\right) f_E. \tag{83}$$

Log-linearization gives

$$\widehat{X}_t = -\frac{D}{\theta C}\left(\widehat{N}_{Et} - \widehat{X}_t\right). \tag{84}$$

Here we use $(1-n_s)\,N f_E = \frac{D}{\theta C}$. From the innovation of the mass of producers, we have

$$\widehat{N}_{t+1} = n_s\widehat{N}_t + (1-n_s)\,\widehat{N}_{Et}. \tag{85}$$

Applying this, we have

$$(1-n_s)\,\widehat{X}_t = -\frac{D}{\theta C}\left[\widehat{N}_{t+1} - n_s\widehat{N}_t - (1-n_s)\,\widehat{X}_t\right]. \tag{86}$$

Thus, we have two dynamic equations

$$\left(1-n_s L^{-1}\right)\widehat{X}_t = (1-n_s)\,\widehat{N}_{t+1}, \tag{87}$$

$$(1-n_s)\left(\frac{\theta C}{D} - 1\right)\widehat{X}_t = -(1-n_s L)\,\widehat{N}_{t+1}, \tag{88}$$

where L is the lag operator. Combining these two equations gives

$$\begin{aligned}(1-n_s)^2(\theta-1)(1-\alpha)\widehat{N}_{t+1} &= -(1-n_sL)\left(1-n_sL^{-1}\right)\widehat{N}_{t+1} \\ &= -\left(1+n_s^2-n_sL-n_sL^{-1}\right)\widehat{N}_{t+1}.\end{aligned} \tag{89}$$

Rearranging it, we have

$$\widehat{N}_{t+2}-2\left[1+\frac{(1-n_s)^2\left[(\theta-1)(1-\alpha)+1\right]}{2n_s}\right]\widehat{N}_{t+1}+\widehat{N}_t=0 \tag{90}$$

Since $1+\frac{(1-n_s)^2[(\theta-1)(1-\alpha)+1]}{2n_s}>1$, we can rewrite the difference equation as

$$\left(1-\mu_NL^{-1}\right)\left(1-\frac{1}{\mu_N}L^{-1}\right)\widehat{N}_t=0, \tag{91}$$

where $0<\mu_N<1$. From the stationarity restriction, we then have

$$\left(1-\frac{1}{\mu_N}L^{-1}\right)\widehat{N}_t=0, \tag{92}$$

or

$$\widehat{N}_{t+1}=\mu_N\widehat{N}_t. \tag{93}$$

That is, the mass of producers is monotonically converges toward the new steady state if the initial N_t is off the new steady state. It also implies that as in the case of a tariff cut, if the initial N_t is the same as the new steady state level, the mass of producers does not change over time.

From the dynamics of $\widehat{X}_t$ we have

$$\widehat{X}_t=\frac{(1-n_s)\mu_N}{1-n_s\mu_N}\widehat{N}_t. \tag{94}$$

Thus, the capital to wage rate ratio is also monotonically converges toward the new steady state if the ratio is initially off the new steady state level. Again, it also implies that if the level of N_t reaches the new steady state level, the capital to wage rate ratio does not change over time.

From the marginal cost in production, we have

$$\begin{aligned}\widehat{MC}_t &= \alpha\widehat{R}_t + (1-\alpha)\widehat{W}_t \\ &= \alpha\left(\widehat{K}_t - \widehat{K}_{t-1}\right) + (1-\alpha)\widehat{W}_t.\end{aligned} \tag{95}$$

The price normalization gives

$$(\theta-1)\widehat{MC}_t = \widehat{N}_t. \tag{96}$$

The capital stock evolves as

$$\begin{aligned}\widehat{K}_t &= \widehat{X}_t + \widehat{W}_t \\ &= \alpha K_{t-1} + \left[\frac{(1-\alpha)(1-n_s)\mu_N}{1-n_s\mu_N} + \frac{1}{\theta-1}\right]\widehat{N}_t,\end{aligned} \tag{97}$$

and the consumption evolves as

$$\widehat{C}_t = \alpha K_{t-1} + \left[\frac{(1-\alpha)(1-n_s)\mu_N}{1-n_s\mu_N} + \frac{1}{\theta-1}\right]\widehat{N}_t. \tag{98}$$

Thus, the capital stock and the consumption evolve based on two state variables, capital stock and the mass of producers available today. It also implies that if the steady state level of N_t is achieved initially as in the case of the iceberg tariff cut, the capital stock evolves monotonically toward the new steady state level with the rate of α only from the capital stock innovation. Thus, the dynamic welfare gains are lower than the static gains in *No-Cost* model.

Trade Growth

The trade share is given as

$$TR_t = \frac{\xi^{1-\theta}}{1+\xi^{1-\theta}}, \tag{99}$$

which does not depends on any variables except the iceberg tariff rate. So the trade share jumps to the new steady state immediately.

B. Fixed Cost Model with Capital

Now, let's look at the model with fixed costs in exporting.

Steady State

In the steady state, we have $R=1$. Again, consumption and capital is always proportional to output

$$K = \frac{\alpha(\theta-1)}{\theta}D, \tag{100}$$
$$C = \left[1-\frac{\alpha(\theta-1)}{\theta}\right]D. \tag{101}$$

From the budget constraint, we have

$$C = W. \tag{102}$$

From the exporting decision together with the price normalization, we have

$$N = \frac{1}{f_X\theta}\left(\frac{D}{C}\right)\left(\frac{\xi^{1-\theta}a_x}{\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_X}\right). \tag{103}$$

From the entry decision, together with the price normalization, we have

$$N = \frac{1}{f_E\theta(1-n_s)}\left(\frac{D}{C}\right)\left(\frac{\overline{\Psi}_T+\left(\frac{1}{\eta}\right)\xi^{1-\theta}\overline{\Psi}_X}{\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_X}\right). \tag{104}$$

Combining these two gives

$$\frac{\xi^{1-\theta}a_x}{f_X} = \frac{\overline{\Psi}_T+\left(\frac{1}{\eta}\right)\xi^{1-\theta}\overline{\Psi}_X}{f_E(1-n_s)}. \tag{105}$$

Long-Run Growth Rates

Log-linearization of the exporting and entry decisions gives

$$\widehat{a}_x = (\theta-1)(1-\lambda_D)\widehat{\xi}, \tag{106}$$

where $\lambda_D = 1-TR = \overline{\Psi}_T/\left(\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_X\right)$, domestic sales to GDP ratio. Thus, a cut in the iceberg cost lowers the exporting threshold and raises the exporter ratio. With this result together with the exporting decision, we have

$$\widehat{N} = (\eta-1)(\theta-1)(1-\lambda_D)\lambda_D\widehat{\xi}. \tag{107}$$

The mass of producers always falls following a cut in iceberg costs. From the price normalization together with the marginal cost in production, we have

$$\widehat{C} = -\left(\frac{1-\lambda_D}{1-\alpha}\right)\widehat{\xi}. \tag{108}$$

Consumption always rises with a tariff cut, and the growth rate is increasing in α once we calibrate the model to match the initial $TR = 1-\lambda_D$. Log-linearization of the trade share of GDP gives

$$\widehat{TR} = -(\theta-1)\lambda_D\left[1+(\eta-1)\lambda_D\right]\widehat{\xi}. \tag{109}$$

With the simplified version of *Fixed Cost* model, the long-run welfare result is exactly the same as *No-Cost* model, whereas the trade growth is greater than *No-Cost* model due to the extensive margin.

Transition Dynamics

Let's log-linearize the system of equations around the new steady state. From the results for the capital-consumption ratio, we have

$$\widehat{D}_t = \widehat{K}_t = \widehat{C}_t. \tag{110}$$

From the capital share equation, we have

$$\widehat{R}_t = \widehat{K}_t - \widehat{K}_{t-1}. \tag{111}$$

The exporting decision is given as

$$W_t f_x = \frac{1}{\theta}\left(\frac{\xi^{1-\theta}a_{xt}}{\overline{\Psi}_T + \xi^{1-\theta}\overline{\Psi}_{Xt}}\right)\left(\frac{D_t}{N_t}\right). \tag{112}$$

Log-linearization after dividing both sides with C_t gives

$$\widehat{N}_t = \widehat{X}_t + \left[1+(\eta-1)(1-\lambda_D)\right]\widehat{a}_{xt}. \tag{113}$$

The entry decision can be rewritten as

$$\left(\frac{W_t}{C_t}\right) = \frac{1}{\theta f_E}\left(\frac{D_{t+1}}{C_{t+1}}\right)\left(\frac{1}{N_{t+1}}\right)\left(\frac{\overline{\Psi}_T + \left(\frac{1}{\eta}\right)\xi^{1-\theta}\overline{\Psi}_{Xt+1}}{\overline{\Psi}_T + \xi^{1-\theta}\overline{\Psi}_{Xt+1}}\right) + n_s\left(\frac{W_{t+1}}{C_{t+1}}\right). \tag{114}$$

Log-linearization gives

$$0 = \widehat{X}_t - n_s \widehat{X}_{t+1} + (1 - n_s) \left[\frac{(\eta - 1)^2 (1 - \lambda_D) \lambda_D}{(\eta - 1) \lambda_D + 1} \widehat{a}_{xt+1} - \widehat{N}_{t+1} \right]. \tag{115}$$

Using the results in (113), we can rewrite it as

$$\widehat{X}_{t+1} - \widehat{X}_t = -\phi_1 \widehat{a}_{xt+1}, \tag{116}$$

where $\phi_1 = \frac{(1-n_s)^2 \eta}{(\eta-1)\lambda_D + 1}$. The budget constraint can be rewritten as

$$1 = \left(\frac{W_t}{C_t} \right) + \frac{1}{\theta} \left(\frac{D_t}{C_t} \right) \left(\frac{\overline{\Psi}_T + \left(\frac{1}{\eta} \right) \xi^{1-\theta} \overline{\Psi}_{Xt}}{\overline{\Psi}_T + \xi^{1-\theta} \overline{\Psi}_{Xt}} \right) - N_{Et} \left(\frac{W_t}{C_t} \right) f_{E.} \tag{117}$$

Log-linearization gives

$$\begin{aligned} 0 \quad = \quad & -\widehat{X}_t + \frac{1}{\theta} \left(\frac{D}{C} \right) \left(\frac{\overline{\Psi}_T + \left(\frac{1}{\eta} \right) \xi^{1-\theta} \overline{\Psi}_X}{\overline{\Psi}_T + \xi^{1-\theta} \overline{\Psi}_X} \right) \frac{(\eta - 1)^2 (1 - \lambda_D) \lambda_D}{(\eta - 1) \lambda_D + 1} \widehat{a}_{xt} \\ & - (1 - n_s) N f_E \left(\widehat{N}_{Et} - \widehat{X}_t \right). \end{aligned} \tag{118}$$

Using

$$\begin{aligned} (1 - n_s) N f_E \quad & = \quad \frac{1}{\theta} \left(\frac{D}{C} \right) \left(\frac{\overline{\Psi}_T + \left(\frac{1}{\eta} \right) \xi^{1-\theta} \overline{\Psi}_X}{\overline{\Psi}_T + \xi^{1-\theta} \overline{\Psi}_X} \right) \\ & = \quad \frac{1}{\theta} \left(\frac{D}{C} \right) \left(\frac{(\eta - 1) \lambda_D + 1}{\eta} \right), \end{aligned} \tag{119}$$

together with (113) and (116), we can rewrite it as

$$\widehat{X}_t = \phi_2 \left(\widehat{a}_{xt} - \widehat{a}_{xt+1} \right), \tag{120}$$

where $\phi_2 = \frac{D}{\theta C \eta (1-n_s)} \left[n_s \eta + (\eta - 1)^2 (1 - \lambda_D) \lambda_D \right]$. Applying this result to (116), we have

$$\phi_2 \left(\widehat{a}_{xt+1} - \widehat{a}_{xt+2} \right) - \phi_2 \left(\widehat{a}_{xt} - \widehat{a}_{xt+1} \right) = -\phi_1 \widehat{a}_{xt+1}. \tag{121}$$

Rewriting it

$$\phi_2 \widehat{a}_{xt+2} - (2\phi_2 + \phi_1) \widehat{a}_{xt+1} + \phi_2 \widehat{a}_{xt} = 0. \tag{122}$$

Dividing both sides with ϕ_2,

$$\widehat{a}_{xt+2} - 2\left(1 + \frac{\phi_1}{2\phi_2}\right) \widehat{a}_{xt+1} + \widehat{a}_{xt} = 0. \tag{123}$$

Since, $1 + \frac{\phi_1}{2\phi_2} > 1$ we can rewrite the 2nd order difference equation as

$$\left(1 - \mu_a L^{-1}\right)\left(1 - \frac{1}{\mu_a} L^{-1}\right) \widehat{a}_{xt} = 0 \tag{124}$$

with $0 < \mu_a < 1$. From the stationarity restriction, we can simplify the equation as

$$\widehat{a}_{xt+1} = \mu_a \widehat{a}_{xt}. \tag{125}$$

Thus, the export threshold and the exporter ratio makes a monotonic transition toward the new steady state. Applying (125) to (120), we have

$$\widehat{X}_t = \phi_2 (1 - \mu_a) \widehat{a}_{xt}, \tag{126}$$
$$\widehat{X}_{t+1} = \mu_a \widehat{X}_t \tag{127}$$

The capital to wage rate ratio also makes a monotonic transition. From (113),

$$\widehat{N}_t = [\phi_2 (1 - \mu_a) + 1 + (\eta - 1)(1 - \lambda_D)] \widehat{a}_{xt}, \tag{128}$$
$$\widehat{N}_{t+1} = \mu_a \widehat{N}_t. \tag{129}$$

The mass of producers also makes a monotonic transition. Contrary to *No-Cost* model, the mass of producers falls following a cut in the tariff rate. Thus, the mass of producers monotonically decreases toward the new steady state, so does the export threshold. The exporter ratio monotonically rises.

From the marginal cost in production and the price normalization,

$$\widehat{MC}_t = \alpha \widehat{R}_t + (1 - \alpha) \widehat{W}_t \tag{130}$$
$$= \alpha \left(\widehat{K}_t - \widehat{K}_{t-1}\right) + (1 - \alpha) \widehat{W}_t,$$
$$(\theta - 1) \widehat{MC}_t = \widehat{N}_t - (\eta - 1)(1 - \lambda_D) \widehat{a}_{xt}. \tag{131}$$

Thus,

$$\begin{aligned}\widehat{K}_t &= \widehat{X}_t + \widehat{W}_t \\ &= \alpha \widehat{K}_{t-1} + \left(\frac{1}{\theta-1}\right)\left\{\frac{1+\phi_2\left(1-\mu_a\right)\left[\left(\theta-1\right)\left(1-\alpha\right)+1\right]}{\phi_2\left(1-\mu_a\right)+1+\left(\eta-1\right)\left(1-\lambda_D\right)}\right\}\widehat{N}_t,\end{aligned} \tag{132}$$

Since K rises but N falls in the long-run, and N_t makes a monotonic transition toward a lower level of new steady state, the economy experiences a overshooting behavior.

The trade share of output is given as

$$TR_t = \frac{\xi^{1-\theta}\overline{\Psi}_{Xt}}{\overline{\Psi}_T + \xi^{1-\theta}\overline{\Psi}_{Xt}}.$$

Log-linearization gives

$$\begin{aligned}\widehat{TR}_t &= \left(1-\eta\right)\widehat{a}_{xt} - \left(1-\lambda_D\right)\left(1-\eta\right)\widehat{a}_{xt} \\ &= -\left(\eta-1\right)\lambda_D\widehat{a}_{xt}.\end{aligned}$$

Thus, the trade share is monotonically increasing toward the new steady state through the extensive margin.

Numerical Example

To illustrate this overshooting behavior, we set $\alpha = 0.34$, $\theta = 5$, $n_s = 0.9$, and $\eta = 1.5$. The fixed cost in exporting and the entry cost is set to be $f_X = 0.040$ and $f_E = 2.668$ to normalize initial $N = 1$, and $n_x = 0.2$ (exporter ratio) with an initial iceberg cost 60 percent. With these parameter values, we have highly persistent transition dynamics with $\mu_a = 0.816$. And, the capital innovation equation is given as

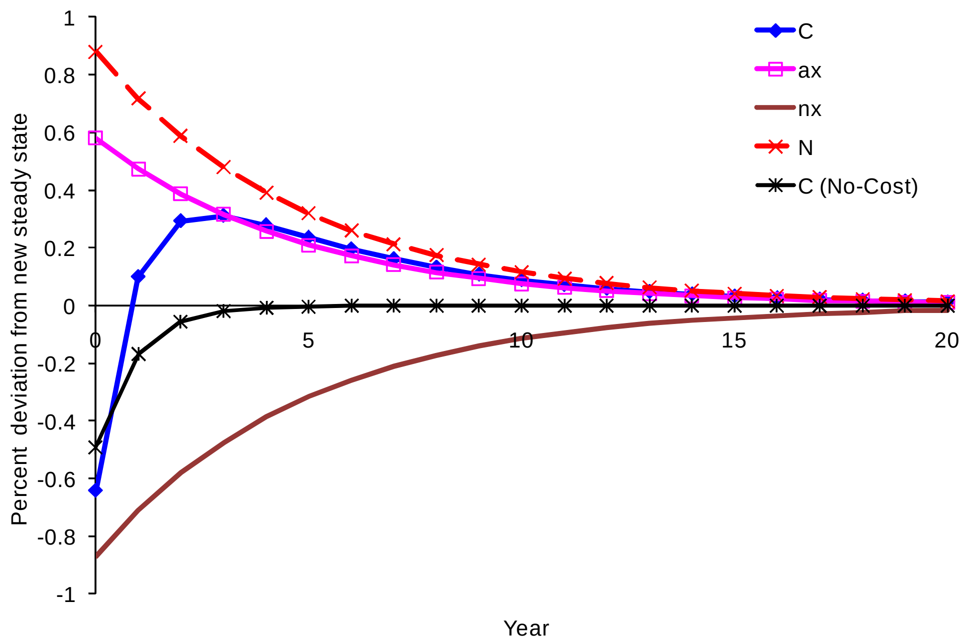
$$\widehat{K}_t = 0.340\widehat{K}_{t-1} + 0.442\widehat{N}_t.$$

Figure 1 shows the responses of variables when the trade cost is cut by 10 percentage points from 60 percent. The mass of producers, export threshold, and the exporter ratio make monotonic transitions toward the new steady state. With the monotonic transition of the mass of producers and the capital accumulation, consumption exhibits a overshooting behavior. The consumption reaches its peak (0.31 percent above the new steady state level) in 4 years after the cut in the tariff rate. After that, consumption slowly converges downward toward the new steady state.[2] Contrary to the fixed model, *No-Cost* model makes a monotonic transition toward the new steady state as the mass of producers

[2] The technical appendix discusses the performance of the log-linearization method. Although the log-linearization slightly overpredicts the overshooting behaviour, the transition dynamics with an exact solution method also exhibits this overshooting behavior.

does not change over time.

Figure 1: Transition Dynamics: Fixed Cost Model



6. Sunk Cost Model

In this section, we show that the welfare and trade growth results in the presence of a sunk cost in exporting are different from those under the fixed cost. Specifically, the trade and the exporter ratio growth rates following a tariff cut are higher in *Sunk Cost* model compared to *Fixed Cost* model. However, the long run welfare gains can be either higher or lower in Sunk Cost model than *Fixed Cost* model depending on the parameter values of the model.

To make the *Sunk Cost* model tractable, we use a simplified version of *Sunk Cost* model; no non-tradable good sector, no capital or material inputs, iceberg tariffs, constant death probability, an iid Pareto distribution $\phi(a) = \eta a^{-1-\eta}$, and zero discount rate, $\beta \to 1$, in a symmetric country steady state. Since the productivity follows an iid shock each period, we assume that producers can export their goods right away upon the payments of fixed costs, f_0 or f_1.

The value of producer with its productivity a and last period exporting status m is given as

$$V^{m'}(a,m) = \frac{1}{\theta}\left(\frac{\theta W}{\theta-1}\right)^{1-\theta} Ca\left(1+m'\xi^{1-\theta}\right) - m'Wf_m + n_s EV_{m'}, \tag{133}$$

where m' is the current exporting status, and EV_m is the expected value of producer with exporting status m. The expected (average) value of producer with exporting status m is given as

$$EV_m = \frac{1}{\theta}\left(\frac{\theta W}{\theta-1}\right)^{1-\theta} C\left(\overline{\Psi}_T + \xi^{1-\theta}\overline{\Psi}_m\right) - n_m W f_m + n_s\left[(1-n_m)EV_0 + n_m EV_1\right], \tag{134}$$

where $\overline{\Psi}_m = \int_{a_m}^{\infty} a\phi(a)\,da$ with a_m being exporting decision threshold for last period exporting status of m, and $n_m = \int_{a_m}^{\infty}\phi(a)\,da$. The difference between the values of average for the last period exporters and non-exporters is given as

$$\begin{aligned} dV &= EV_1 - EV_0 \qquad (135)\\ &= \frac{1}{\theta}\left(\frac{\theta W}{\theta-1}\right)^{1-\theta} C\xi^{1-\theta}\left(\overline{\Psi}_1 - \overline{\Psi}_0\right) - \left(n_1 W f_1 - n_0 W f_0\right) + n_s\left(n_1 - n_0\right)dV. \end{aligned}$$

The marginal exporters satisfy

$$W f_m = \frac{1}{\theta}\left(\frac{\theta W}{\theta-1}\right)^{1-\theta} C\xi^{1-\theta} a_m + n_s dV. \tag{136}$$

Applying this condition to dV,

$$\begin{aligned} dV &= \frac{1}{\theta}\left(\frac{\theta W}{\theta-1}\right)^{1-\theta} C\xi^{1-\theta}\left[\left(\overline{\Psi}_1 - n_1 a_1\right) - \left(\overline{\Psi}_0 - n_0 a_0\right)\right] \qquad (137)\\ &= \frac{1}{\theta}\left(\frac{\theta W}{\theta-1}\right)^{1-\theta} C\xi^{1-\theta}\left(\frac{1}{\eta}\right)\left(\overline{\Psi}_1 - \overline{\Psi}_0\right). \end{aligned}$$

Here, the last equation uses the Pareto distribution $\phi(a) = \eta a^{-1-\eta}$. Rewriting the exporting decisions, we have

$$W f_m = \frac{1}{\theta}\left(\frac{\theta W}{\theta-1}\right)^{1-\theta} C\xi^{1-\theta}\left[a_m + \left(\frac{n_s}{\eta}\right)\left(\overline{\Psi}_1 - \overline{\Psi}_0\right)\right]. \tag{138}$$

The measures of producers are given as

$$N_E = (1-n_s)N, \tag{139}$$

$$N_1 = n_s n_1 N_1 + n_0\left(N_E + n_s N_0\right), \tag{140}$$

$$N_0 = n_s\left(1-n_1\right)N_1 + \left(1-n_0\right)\left(N_E + n_s N_0\right). \tag{141}$$

Thus, we have exporter and non-exporter ratios as

$$\frac{N_1}{N} = \frac{n_0}{1 - n_s (n_1 - n_0)}, \tag{142}$$

$$\frac{N_0}{N} = \frac{1 - n_0 - n_s (n_1 - n_0)}{1 - n_s (n_1 - n_0)}. \tag{143}$$

The measures of continuing and new exporters are given as

$$N_{11} = \frac{n_s n_1 n_0}{1 - n_s (n_1 - n_0)} N,$$

$$N_{01} = \frac{n_0 (1 - n_s n_1)}{1 - n_s (n_1 - n_0)} N.$$

The price normalization gives

$$1 = \left(\frac{\theta W}{\theta - 1} \right)^{1-\theta} N \left(\overline{\Psi}_T + \xi^{1-\theta} \overline{\Psi}_X \right), \tag{144}$$

where

$$\begin{aligned} \overline{\Psi}_X &= \left(\frac{n_s N_1}{N} \right) \overline{\Psi}_1 + \left(\frac{n_s N_0 + N_E}{N} \right) \overline{\Psi}_0 \\ &= \overline{\Psi}_0 + \left(\frac{n_s N_1}{N} \right) \left(\overline{\Psi}_1 - \overline{\Psi}_0 \right). \end{aligned} \tag{145}$$

The expected value of last period non-exporter is given as

$$EV_0 = \frac{1}{\theta (1 - n_s)} \left(\frac{\theta W}{\theta - 1} \right)^{1-\theta} C \left(\overline{\Psi}_T + \frac{1}{\eta} \xi^{1-\theta} \overline{\Psi}_0 \right). \tag{146}$$

The entry condition is given as

$$W f_E = \frac{1}{\theta (1 - n_s)} \left(\frac{\theta W}{\theta - 1} \right)^{1-\theta} C \left(\overline{\Psi}_T + \frac{1}{\eta} \xi^{1-\theta} \overline{\Psi}_0 \right).$$

The total profit of firms is given as

$$\Pi = \frac{C}{\theta}.$$

The total costs in creating firms are given as

$$\left(1-n_s\right)NWf_E=\frac{1}{\theta}\left(\frac{\theta W}{\theta-1}\right)^{1-\theta}CN\left(\overline{\Psi}_T+\frac{1}{\eta}\xi^{1-\theta}\overline{\Psi}_0\right). \tag{147}$$

The total fixed cost payments for exporting are given as

$$\begin{aligned} N_{11}Wf_1+N_{01}Wf_0 &= \frac{1}{\theta}\left(\frac{\theta W}{\theta-1}\right)^{1-\theta}C\xi^{1-\theta}N \qquad (148)\\ & \left\{n_s n_1\frac{N_1}{N}\left[a_1+\left(\frac{n_s}{\eta}\right)\left(\overline{\Psi}_1-\overline{\Psi}_0\right)\right]+n_0\left(1-\frac{n_s N_1}{N}\right)\left[a_0+\left(\frac{n_s}{\eta}\right)\left(\overline{\Psi}_1-\overline{\Psi}_0\right)\right]\right\}. \end{aligned}$$

The budget constraint is given as

$$\begin{aligned} C &= WL+\Pi-\left(1-n_s\right)NWf_E-N_{11}Wf_1-N_{01}Wf_0 \qquad (149)\\ &= WL. \end{aligned}$$

Exporting Thresholds

From the exporting decisions, we have

$$\frac{f_0}{f_1}=\frac{a_0+\left(\frac{n_s}{\eta}\right)\left(\overline{\Psi}_1-\overline{\Psi}_0\right)}{a_1+\left(\frac{n_s}{\eta}\right)\left(\overline{\Psi}_1-\overline{\Psi}_0\right)}. \tag{150}$$

Log-linearization gives

$$\frac{a_0\widehat{a}_0-\frac{n_s(\eta-1)}{\eta}\left(\overline{\Psi}_1\widehat{a}_1-\overline{\Psi}_0\widehat{a}_0\right)}{a_0+\left(\frac{n_s}{\eta}\right)\left(\overline{\Psi}_1-\overline{\Psi}_0\right)}=\frac{a_1\widehat{a}_1-\frac{n_s(\eta-1)}{\eta}\left(\overline{\Psi}_1\widehat{a}_1-\overline{\Psi}_0\widehat{a}_0\right)}{a_1+\left(\frac{n_s}{\eta}\right)\left(\overline{\Psi}_1-\overline{\Psi}_0\right)}. \tag{151}$$

Rearranging the equations,

$$\begin{aligned} f_1\left[a_0\widehat{a}_0-\frac{n_s\left(\eta-1\right)}{\eta}\left(\overline{\Psi}_1\widehat{a}_1-\overline{\Psi}_0\widehat{a}_0\right)\right] &= f_0\left[a_1\widehat{a}_1-\frac{n_s\left(\eta-1\right)}{\eta}\left(\overline{\Psi}_1\widehat{a}_1-\overline{\Psi}_0\widehat{a}_0\right)\right], \qquad (152)\\ \left[f_1a_0-\left(\frac{n_s\left(\eta-1\right)}{\eta}\right)\overline{\Psi}_0\left(f_0-f_1\right)\right]\widehat{a}_0 &= \left[f_0a_1-\frac{n_s\left(\eta-1\right)}{\eta}\overline{\Psi}_1\left(f_0-f_1\right)\right]\widehat{a}_1. \qquad (153) \end{aligned}$$

Thus, we have

$$\widehat{a}_1=z\widehat{a}_0, \tag{154}$$

where

$$z=\frac{f_1a_0-\frac{n_s(\eta-1)}{\eta}\overline{\Psi}_0\left(f_0-f_1\right)}{f_0a_1-\frac{n_s(\eta-1)}{\eta}\overline{\Psi}_1\left(f_0-f_1\right)} \tag{155}$$

Since $f_1a_0=f_0a_1+\left(f_0-f_1\right)\left(\frac{n_s}{\eta}\right)\left(\overline{\Psi}_1-\overline{\Psi}_0\right)$ and $\overline{\Psi}_1>\overline{\Psi}_0$ with $f_0>f_1$, we have $f_1a_0>f_0a_1$ and $f_1a_0-\frac{n_s(\eta-1)}{\eta}\overline{\Psi}_0\left(f_0-f_1\right)>f_0a_1-\frac{n_s(\eta-1)}{\eta}\overline{\Psi}_1\left(f_0-f_1\right)$. We also have

$$f_0a_1-\frac{n_s\left(\eta-1\right)}{\eta}\overline{\Psi}_1\left(f_0-f_1\right)=f_0a_1\left[1-n_s\left(\frac{f_0-f_1}{f_0}\right)a_1^{-\eta}\right]>0. \tag{156}$$

Thus, $z>1$. That is the long-run response of a_1 is greater than a_0 and these thresholds move in the same direction.

Marginal Change of a_0.

Now, we can use the exporting decision and the entry decision to get the long-run change in a_0. We have

$$\frac{f_E\left(1-n_s\right)}{f_0}=\frac{\overline{\Psi}_T+\frac{1}{\eta}\xi^{1-\theta}\overline{\Psi}_0}{\xi^{1-\theta}\left[a_0+\left(\frac{n_s}{\eta}\right)\left(\overline{\Psi}_1-\overline{\Psi}_0\right)\right]}. \tag{157}$$

Log-linearization gives

$$\frac{\xi^{1-\theta}\overline{\Psi}_0}{\eta\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_0}\left[\left(1-\theta\right)\widehat{\xi}+\left(1-\eta\right)\widehat{a}_0\right]=\left(1-\theta\right)\widehat{\xi}+\frac{a_0\widehat{a}_0-\frac{n_s(\eta-1)}{\eta}\left(\overline{\Psi}_1\widehat{a}_1-\overline{\Psi}_0\widehat{a}_0\right)}{a_0+\left(\frac{n_s}{\eta}\right)\left(\overline{\Psi}_1-\overline{\Psi}_0\right)}. \tag{158}$$

Sorting this out, we have

$$\begin{aligned}\left(\theta-1\right)\eta\overline{\Psi}_T\left[1+\left(\frac{n_s}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)\right]\widehat{\xi} \quad &= \quad \left(\eta-1\right)\left[1+\left(\frac{n_s}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)\right]\xi^{1-\theta}\overline{\Psi}_0\widehat{a}_0 \\ &\quad +\left(\eta\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_0\right)\left[\widehat{a}_0-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\widehat{a}_1-\frac{\overline{\Psi}_0}{a_0}\widehat{a}_0\right)\right].\end{aligned} \tag{159}$$

Rearranging the right-hand-side,

$$\begin{aligned}RHS \quad &= \quad \eta\left\{\overline{\Psi}_T\left[1-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1z-\overline{\Psi}_0}{a_0}\right)\right]\right. \\ &\quad \left.+\xi^{1-\theta}\overline{\Psi}_0\left[1+\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1z-\overline{\Psi}_0}{a_0}\right)\right]\right\}\widehat{a}_0.\end{aligned} \tag{160}$$

Thus, we have

$$\widehat{a}_0 = \frac{(\theta-1)\left[1+\left(\frac{n_s}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)\right]}{\frac{\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_0}{\overline{\Psi}_T} - n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1 z-\overline{\Psi}_0}{a_0}\right) - \frac{\xi^{1-\theta}\overline{\Psi}_0}{\overline{\Psi}_T}\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)(z-1)}\widehat{\xi}. \tag{161}$$

In *Fixed Cost* model, $f_0 = f_1$, we have $\overline{\Psi}_1 = \overline{\Psi}_0 = \overline{\Psi}_X$ and $z = 1$. Thus, we have $\left(\widehat{a}_0/\widehat{\xi}\right)|_{f_0=f_1} = (\theta-1)\lambda_D$. For any $f_0 > f_1$ under sunk costs in exporting, we have $\overline{\Psi}_1 > \overline{\Psi}_X > \overline{\Psi}_0$ and $z > 1$. So, $\lambda_D\left(\frac{\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_0}{\overline{\Psi}_T}\right) < 1$, and the denominator is less than $1/\lambda_D$. This implies that the response of $\widehat{a}_0$ to $\widehat{\xi}$ is greater than the fixed model case, $\left(\widehat{a}_0/\widehat{\xi}\right)|_{f_0>f_1} > \left(\widehat{a}_0/\widehat{\xi}\right)|_{f_0=f_1}$, provided that $\widehat{a}_0/\widehat{\xi} > 0$. To check the sign of the response, we can rearrange the denominator of the equation as

$$1 - n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)z + n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_0}{a_0}\right) + \frac{\xi^{1-\theta}\overline{\Psi}_0}{\overline{\Psi}_T}\left[1-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)(z-1)\right]. \tag{162}$$

We have $z > 1$, $z > (z-1)/\eta$ and

$$\begin{aligned} 1 - n_s\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}z &= 1 - n_s\frac{a_1^{1-\eta}}{a_0}\left[\frac{f_1 a_0 - \frac{n_s(\eta-1)}{\eta}\overline{\Psi}_0(f_0-f_1)}{f_0 a_1 - \frac{n_s(\eta-1)}{\eta}\overline{\Psi}_1(f_0-f_1)}\right] \\ &= \frac{1 - n_s a_1^{-\eta} + n_s^2 a_0^{-\eta} a_1^{-\eta}\left(\frac{f_0-f_1}{f_0}\right)}{1 - n_s a_1^{-\eta}\left(\frac{f_0-f_1}{f_0}\right)} > 0, \end{aligned} \tag{163}$$

since $a_1^{-\eta} < 1$ and $f_0 > f_1$. Thus, the marginal effect is positive, $\widehat{a}_0/\widehat{\xi} > 0$.

Consumption

From the entry decision together with the budget constraint, we have

$$C^{(\theta-1)} = \frac{L}{f_E\theta(1-n_s)}\left(\frac{\theta}{\theta-1}\right)^{1-\theta}\left(\overline{\Psi}_T + \frac{1}{\eta}\xi^{1-\theta}\overline{\Psi}_0\right). \tag{164}$$

Log-linearization gives

$$(\theta-1)\widehat{C} = -\left(\frac{\xi^{1-\theta}\overline{\Psi}_0}{\eta\overline{\Psi}_T + \xi^{1-\theta}\overline{\Psi}_0}\right)\left[(\theta-1)\widehat{\xi} + (\eta-1)\widehat{a}_0\right]. \tag{165}$$

For $1+\left(\frac{\eta-1}{\theta-1}\right)\frac{\widehat{a}_0}{\widehat{\xi}}$ we have

$$\begin{aligned}
& 1+\left(\frac{\eta-1}{\theta-1}\right)\frac{\widehat{a}_0}{\widehat{\xi}} \\
= & \left\{1-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1 z-\overline{\Psi}_0}{a_0}\right)+(\eta-1)\left[1+\left(\frac{n_s}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)\right]\right. \\
& \left.+\frac{\xi^{1-\theta}\overline{\Psi}_0}{\overline{\Psi}_T}\left[1+\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1 z-\overline{\Psi}_0}{a_0}\right)\right]\right\} \\
& \left\{1-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1 z-\overline{\Psi}_0}{a_0}\right)\right. \\
& \left.+\frac{\xi^{1-\theta}\overline{\Psi}_0}{\overline{\Psi}_T}\left[1+\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1 z-\overline{\Psi}_0}{a_0}\right)\right]\right\}^{-1}.
\end{aligned} \tag{166}$$

The numerator can be reorganized as

$$\begin{aligned}
NUM &= \eta\left[1-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}(z-1)\right] \\
&\quad +\frac{\xi^{1-\theta}\overline{\Psi}_0}{\overline{\Psi}_T}\left[1-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}(z-1)\right] \\
&= \left(\frac{\eta\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_0}{\overline{\Psi}_T}\right)\left[1-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}(z-1)\right].
\end{aligned} \tag{167}$$

The denominator can be reorganized as

$$\begin{aligned}
DEN &= 1-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1 z-\overline{\Psi}_0}{a_0}\right) \\
&\quad +\frac{\xi^{1-\theta}\overline{\Psi}_0}{\overline{\Psi}_T}\left[1+\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1 z-\overline{\Psi}_0}{a_0}\right)\right] \\
&= 1-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)(z-1)-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right) \\
&\quad +\frac{\xi^{1-\theta}\overline{\Psi}_0}{\overline{\Psi}_T}\left[1-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)(z-1)\right].
\end{aligned} \tag{168}$$

Thus, we have

$$1+\left(\frac{\eta-1}{\theta-1}\right)\frac{\widehat{a}_0}{\widehat{\xi}}=\frac{\left(\eta\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_0\right)/\overline{\Psi}_T}{\frac{\xi^{1-\theta}\overline{\Psi}_0}{\overline{\Psi}_T}+\left[\frac{1-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)(z-1)-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)}{1-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}(z-1)}\right]}. \tag{169}$$

For the change in consumption, we have

$$\begin{aligned}\frac{\widehat{C}}{\widehat{\xi}} &= -\frac{\xi^{1-\theta}\overline{\Psi}_0}{\eta\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_0}\left[1+\left(\frac{\eta-1}{\theta-1}\right)\frac{\widehat{a}_0}{\widehat{\xi}}\right] \\ &= -\left\{1+\frac{\overline{\Psi}_T}{\xi^{1-\theta}\overline{\Psi}_0}\left[\frac{1-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)(z-1)-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)}{1-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}(z-1)}\right]\right\}^{-1}.\end{aligned} \tag{170}$$

Clearly,

$$0<\frac{1-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)(z-1)-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)}{1-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}(z-1)}\le 1 \tag{171}$$

for $f_0>f_1$. In *Fixed Cost* model, $f_0=f_1$, we have $z=1$ and $\overline{\Psi}_0=\overline{\Psi}_1=\overline{\Psi}_X$. Thus, we have

$$\begin{aligned}\widehat{C}|_{f_0=f_1} &= -\left(\frac{\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_0}{\xi^{1-\theta}\overline{\Psi}_0}\right)^{-1}\widehat{\xi} \\ &= -(1-\lambda_D)\,\widehat{\xi}.\end{aligned} \tag{172}$$

In *Sunk Cost* model, we have

$$\begin{aligned}\widehat{C} &= -\left\{1+\frac{\overline{\Psi}_T}{\xi^{1-\theta}\overline{\Psi}_0}\left[\frac{1-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)(z-1)-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)}{1-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}(z-1)}\right]\right\}^{-1}\widehat{\xi} \\ &= -\frac{\xi^{1-\theta}\overline{\Psi}_X}{\xi^{1-\theta}\overline{\Psi}_X+\overline{\Psi}_T\left(\frac{\overline{\Psi}_X}{\overline{\Psi}_0}\right)\left[\frac{1-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)(z-1)-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)}{1-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}(z-1)}\right]}\widehat{\xi}.\end{aligned} \tag{173}$$

Let $h=\left(\frac{\overline{\Psi}_X}{\overline{\Psi}_0}\right)\left[\frac{1-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)(z-1)-n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)}{1-\left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}(z-1)}\right]$. If $h>1$, then, $\widehat{C}<(1-\lambda_D)\,\widehat{\xi}$. That is the long-run growth rate of consumption under sunk costs is less that the prediction from *Fixed Cost* model. From the definition of $\overline{\Psi}_X$, we have

$$\frac{\overline{\Psi}_X}{\overline{\Psi}_0}=1+n_sn_x\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{\overline{\Psi}_0}\right). \tag{174}$$

For n_x we have

$$
\begin{aligned}
n_x &= \frac{n_0}{1 - n_s (n_1 - n_0)} \\
&= \frac{\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_0}{a_0}}{1 - n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_1} - \frac{\overline{\Psi}_0}{a_0}\right)} \\
&= \frac{\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_0}{a_0}}{1 - n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right) - n_s\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}\left(\frac{a_0}{a_1} - 1\right)}.
\end{aligned} \tag{175}
$$

So, we can rewrite $\overline{\Psi}_X/\overline{\Psi}_0$ as

$$
\begin{aligned}
\frac{\overline{\Psi}_X}{\overline{\Psi}_0} &= 1 + \frac{n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)}{1 - n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right) - n_s\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}\left(\frac{a_0}{a_1} - 1\right)} \\
&= \frac{1 - n_s\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}\left(\frac{a_0}{a_1} - 1\right)}{1 - n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right) - n_s\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}\left(\frac{a_0}{a_1} - 1\right)}.
\end{aligned} \tag{176}
$$

Now, we have

$$
\begin{aligned}
h = &\left[\frac{1 - n_s\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}\left(\frac{a_0}{a_1} - 1\right)}{1 - n_s\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}\left(\frac{a_0}{a_1} - 1\right) - n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)}\right] \\
&\cdot\left[\frac{1 - n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1}{a_0}\right)(z-1) - n_s\left(\frac{\eta-1}{\eta}\right)\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{a_0}\right)}{1 - \left(\frac{n_s}{\eta}\right)\left(\frac{\eta-1}{\eta}\right)\frac{\overline{\Psi}_1}{a_0}(z-1)}\right].
\end{aligned} \tag{177}
$$

Thus, we have

$$
\begin{aligned}
\widehat{C} &= -\frac{\xi^{1-\theta}\overline{\Psi}_X}{h\overline{\Psi}_T + \xi^{1-\theta}\overline{\Psi}_X}\widehat{\xi} \\
&= -\frac{1-\lambda_D}{1 + (h-1)\lambda_D}\widehat{\xi},
\end{aligned} \tag{178}
$$

The long-run growth rate of consumption under *Sunk Cost* model lower (higher) than that under *Fixed Cost* model if $h > 1$ (< 1).

Trade Growth

The trade share is given as

$$TR=\frac{\xi^{1-\theta}\overline{\Psi}_X}{\overline{\Psi}_T+\xi^{1-\theta}\overline{\Psi}_X}. \tag{179}$$

The growth rate of trade share is given as

$$\begin{aligned}\widehat{TR} &= (1-\theta)\,\widehat{\xi}+\widehat{\overline{\Psi}}_X-(1-\lambda_D)\left[(1-\theta)\,\widehat{\xi}+\widehat{\overline{\Psi}}_X\right] \\ &= \lambda_D\left[(1-\theta)\,\widehat{\xi}+\widehat{\overline{\Psi}}_X\right].\end{aligned} \tag{180}$$

The growth rate of $\overline{\Psi}_X$, $\widehat{\overline{\Psi}}_X$ is given as

$$\overline{\Psi}_X\widehat{\overline{\Psi}}_X=\overline{\Psi}_0\widehat{\overline{\Psi}}_0+n_sn_x\left(\overline{\Psi}_1-\overline{\Psi}_0\right)\widehat{n}_x+n_sn_x\left(\overline{\Psi}_1\widehat{\overline{\Psi}}_1-\overline{\Psi}_0\widehat{\overline{\Psi}}_0\right). \tag{181}$$

Rewriting it, we have

$$\begin{aligned}\widehat{\overline{\Psi}}_X &= \left(\frac{\overline{\Psi}_0}{\overline{\Psi}_X}\right)\widehat{\overline{\Psi}}_0+n_sn_x\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{\overline{\Psi}_X}\right)\widehat{n}_x+n_sn_x\left[\left(\frac{\overline{\Psi}_1}{\overline{\Psi}_X}\right)\widehat{\overline{\Psi}}_1-\left(\frac{\overline{\Psi}_0}{\overline{\Psi}_X}\right)\widehat{\overline{\Psi}}_0\right] \\ &= n_sn_x\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{\overline{\Psi}_X}\right)\widehat{n}_x-(\eta-1)\,\widehat{a}_0-(\eta-1)\,n_sn_x\left(\frac{\overline{\Psi}_1}{\overline{\Psi}_X}\right)(z-1)\,\widehat{a}_0.\end{aligned} \tag{182}$$

For the growth rate of exporter ratio, we have

$$\begin{aligned}\widehat{n}_x &= \widehat{n}_0+\frac{n_s\left(n_1\widehat{n}_1-n_0\widehat{n}_0\right)}{1-n_s\left(n_1-n_0\right)} \\ &= -\eta\left[1+\frac{n_s\left(n_1z-n_0\right)}{1-n_s\left(n_1-n_0\right)}\right]\widehat{a}_0.\end{aligned} \tag{183}$$

Since $\widehat{a}_0/\widehat{\xi}$ is greater under *Sunk Cost* model compared to *Fixed Cost* model, $\left(\widehat{a}_0/\widehat{\xi}\right)|_{f_0>f_1}>\left(\widehat{a}_0/\widehat{\xi}\right)|_{f_0=f_1}>0$, and $\frac{n_s(n_1z-n_0)}{1-n_s(n_1-n_0)}>0$ under *Sunk Cost* model whereas it is 0 in *Fixed Cost* model, $\left|\widehat{n}_x/\widehat{\xi}\right|_{f_0>f_1}>\left|\widehat{n}_x/\widehat{\xi}\right|_{f_0=f_1}$. That is the exporter ratio rises more compared to the fixed cost model following a tariff cut.

The growth rate of the trade share is given as

$$\begin{aligned}\widehat{TR} &= -\lambda_D\left[(\theta-1)\,\widehat{\xi}+(\eta-1)\,\widehat{a}_0\right] \\ &\quad -\lambda_D\left\{\eta n_sn_x\left(\frac{\overline{\Psi}_1-\overline{\Psi}_0}{\overline{\Psi}_X}\right)\left[1+\frac{n_s\left(n_1z-n_0\right)}{1-n_s\left(n_1-n_0\right)}\right]+(\eta-1)\,n_sn_x\left(\frac{\overline{\Psi}_1}{\overline{\Psi}_X}\right)(z-1)\right\}\widehat{a}_0.\end{aligned} \tag{184}$$

Since $\widehat{a}_0/\widehat{\xi}$ is greater under *Sunk Cost* model compared to *Fixed Cost* model, and the multiplier in the second term is always positive under *Sunk Cost* model whereas it is zero under *Fixed Cost* model, the trade growth is higher under *Sunk Cost* model compared to *Fixed Cost* model.